CHE392-Chemical Engineering Laboratory I

Online Education Term

5B- UNSTEADY STATE HEAT TRANSFER

THEORY

Heat transfermay occurby any one ormore of the three basic mechanism of heat transfer:

Conduction: The heat is conducted by the transfer of the energy of motion between adjacent molecules. In conduction, enegy can also be transferred by free electrons.

The transfer of heat by conduction follows this basic equation and is written as Fourier's Law for heat conduction in fluids or solids:

$$Q=-k*A*(dT/dx)$$

Q:Heat transfer rate in W

A: Cross-sectional area normal to the direction of flow of heat in m^2

T: Temperature in K

x: Distance in m

k: Thermal conductivity in W/m.K

2. Convection: The transfer of heat by convection implies the transfer of heat by bulk transport and mixing of macroscopc elements of warmer portionswith cooler portions of a gas or liquid.

$$Q=h*A*(Tf-Tw)$$

Q: Heat transfer rate in W

A: Area in m^2

Tf: Average ot bulk temperature of fluid flowing past in K

Tw:Temperature of the solid surface in K

h: Convective heat transfer coefficient $W/m^2.K$

3. Radiation: Radiation isthe transfer of energy through space by means of electromagnetic waves in much the same way as electromagnetic light waves transfer light.

The net heat of absorption is then, by the Stefan-Boltzmann equaiton:

$$Q=\sigma*A*T^4$$

Q:Heat flow in W

A: Surface area of body in m^2

σ: Constant $5,676x10^{-8}$ $W/m^2.K^4$

T: Temperature of the black body in K

Geankoplis, C. J. (2003). *Transport processes and separation process principles:(includes unit operations)*. Prentice Hall Professional Technical Reference.

Dimensionless Number

- **Nusselt Number:** Nu is used to relate data for the heat transfer coefficient h to the thermal conductivity k of the fluid and a characteristic dimension D.

$$Nu=\frac{hD}{k}$$

- **Prandtl Number:** Pr is the ratio of the shear component of diffusivity for momentum μ/ρ to the diffusivity for heat k/ρ*cp.

$$Pr=\frac{cp*\mu}{k}$$

- **Reynold Number:** Ratio of kinetic or inertial forces to the viscous forces in the fluid stream.

$$Re=\frac{D*V*\rho}{\mu}$$

- **Biot Number:** Biot number compares the relative valuesof internal conduction resistance and surface convective resistance to heat transfer.

$$\mathrm{Bi}=\frac{h\,l}{k}$$

- **Grashof Number:** Ratio of forcing (buoyancy) forces to restraining (viscous) forces.

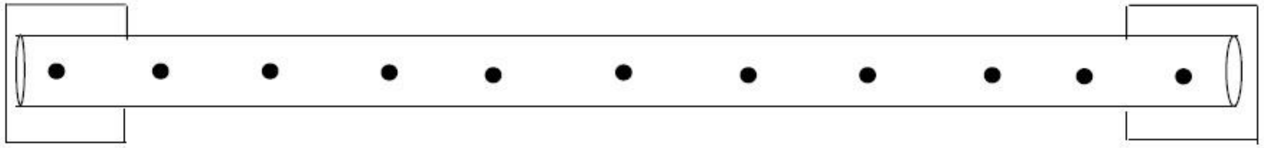
$$\mathrm{Gr}=\frac{g\,\beta\,(T_w-T_\infty)\,D^3}{k}$$

Objective

- The purpose is to examine the temperature distribution along the brass rod maintained at two fixed temperatures and the heat loss from this rod.

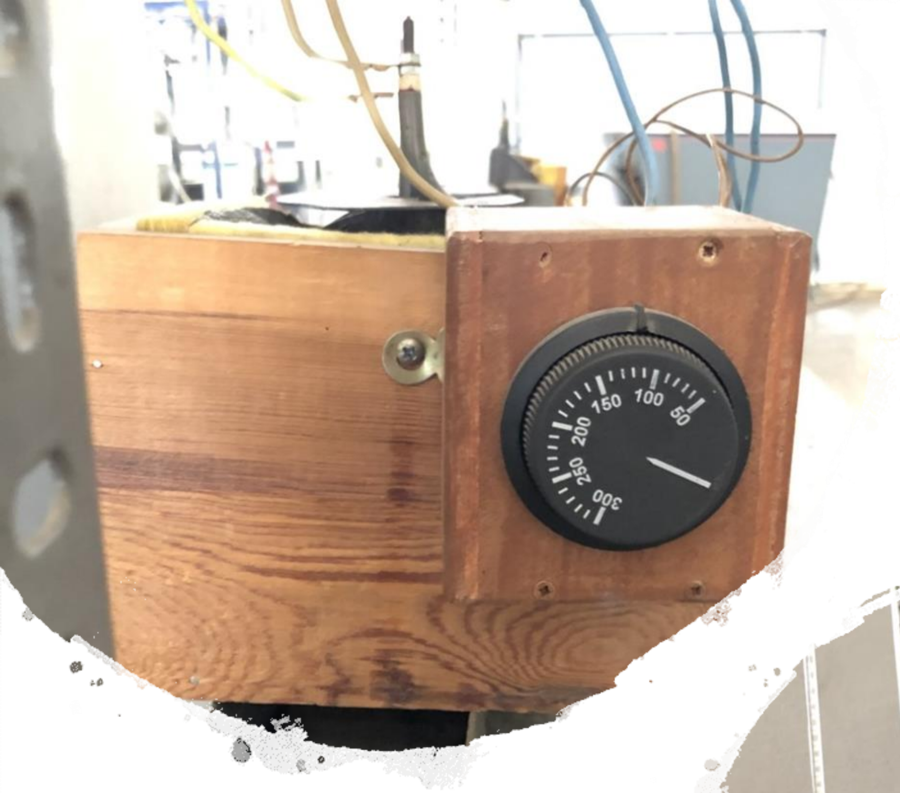
- In this experiment, we will study processes in which the temperature at any given point in the system changes with time, that is, heat transfer is unsteady state or transient.

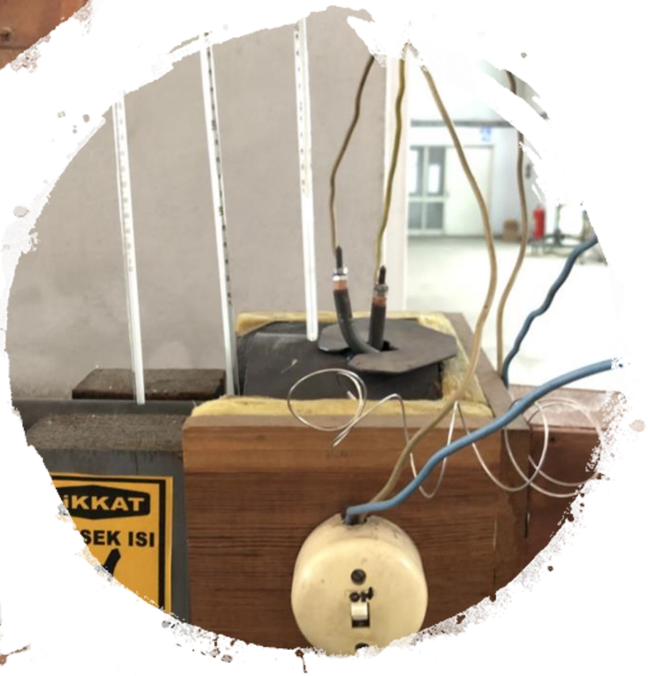
Experimental Setup



- It is a cylindrical rod that is heated by electric power from its end sides and placed in a thermometer in various positions along its length.

- 11 thermometers are placed on the approximately 1 meter long brass rod.





- The two ends of the cylindrical brass rod are heated to 150 ºC with electrical resistances and are kept constant at this temperature.
- Every 10 minutes, the temperatures are read from the thermometers placed along the rod at the same time.
- This process lasts up to 10 minutes after the thermometers in the rod have reached 150 ºC.
- The obtained data are recorded in the table.
- The ambient temperature is also measured.

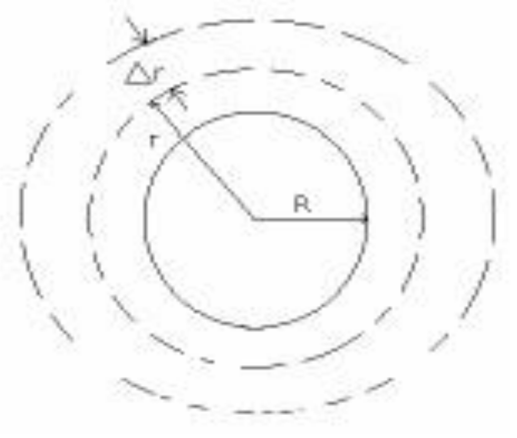
HEAT TRANSFER THROUGH STAGNANT FLUID ENVIRONMENT FROM CYLINDER SURFACE

Assumptions:

Cylinder outer surface temperature T_w = constant

Steady state

In the case of ∂r = 785R T = T_∞



$$\frac{(2\pi rL)q|_r - (2\pi rL)q|_{r+\Delta r}}{(2\pi r)\Delta r} = 0$$

$$\lim_{\Delta r \to 0}\left(\frac{r\,q|_r - r\,q|_{r+\Delta r}}{\Delta r}\right) = 0 \Rightarrow -\frac{d(qr)}{dr} = 0 \qquad -q = \frac{c_1}{r} \qquad q = -hA\frac{dT}{dr} \rightarrow -\left(-hA\frac{dT}{dr}\right) = \frac{c_1}{r} \qquad \frac{dT}{dr} = \frac{c_1}{hAr} \qquad T = c'\ln r + c_2$$

Boundary condition

$T = Tw \quad r = R$

$T = T_\infty \quad r = 785R$

$$T_w = c'\ln R + c_2$$

$$T_\infty = c'\ln 785R + c_2$$

$$T_w - T_\infty = c'(\ln R - \ln 785R)$$

$$T_w - T_\infty = c'\ln\frac{R}{785R} \Rightarrow c' = \frac{T_w - T_\infty}{\ln\left(\frac{R}{785R}\right)}$$

$$c' = \frac{(T_w - T_\infty)}{-6{,}666}$$

$$q\big|_{r=R} = ?$$

$$q\big|_{r=R} = (2\pi RL)(-k\frac{dT}{dr})\big|_{r=R} = -2\pi RLk\frac{C_1}{r}\bigg|_{r=R} = -2\pi RLk\frac{(T_w - T_\infty)}{(-6{,}666)R} = \frac{\pi Lk}{3{,}333}(T_w - T_\infty)$$

$$q\big|_{r=R} = \frac{\pi Lk}{3{,}333}(T_w - T_\infty) \equiv h(T_w - T_\infty)(2\pi RL) \Rightarrow \frac{k}{3{,}333} = hD \Rightarrow \frac{hD}{k} = 0{,}3 \quad \Rightarrow \quad \text{Nu}=0{,}3$$

HEAT TRANSFER BY NATURAL CONVECTION FROM HORIZONTAL CYLINDER

- Average temperature of the cylindrical bar Tw
- Ambient temperature T∞

→ Film temperature (Tf)

300K	350K
ρ = 1.1774 kg/m^3	ρ = 0.9980 kg/m^3
C_p=1.0057 kj/kg.K	C_p=1.0090 kj/kg.K
μ =1.8462×10^{-5}kg/m.s	μ =2.075×10^{-5}kg/m.s
ν =15.69×10^{-6}m^2/s	ν =20.76×10^{-6}m^2/s
k= 0.02624 W/m.K	k= 0.03003 W/m.K
Pr= 0.708	Pr= 0.697

Yalçın H, Gürü M, " Mühendislik Termodinamiği " , 2. Baskı, Palme Yayıncılık, 457-486, (2004).

Nusselt Number $Nu = \frac{hD}{k}$

Nu=cRa^n

Prandtl Number $\Pr = \frac{C_p \mu}{k}$

Grashof_Number $Gr = \frac{g\beta(T_w - T_\infty)D^3}{\gamma^2}$

Rayling Number Ra=Gr.Pr

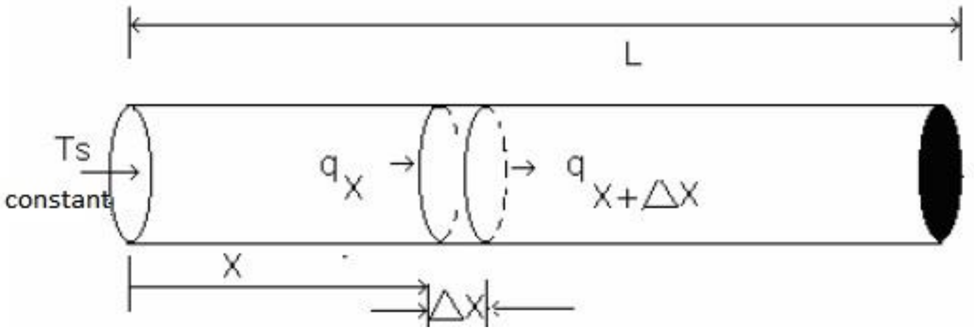
Gr → Ra → Nu → h → q

Ra	c	n
10^{-10}-10^{-2}	0.675	0.058
10^{-2}-10^{2}	1.02	0.148
10^{2}-10^{4}	0.85	0.188
10^{4}-10^{7}	0.48	0.25
10^{7}-10^{12}	0.125	0.333

HEAT LOSS FROM CYLINDRICAL FIN (STEADY STATE)

$$\theta = (T - T_\infty); \quad m^2 = \frac{hP}{kA} \quad \Rightarrow \quad \frac{\partial^2 \theta}{\partial x^2} = m^2 \theta$$

$$\theta = C_1 e^{-mx} + C_2 e^{+mx} \qquad C_1 = \theta_s \frac{e^{mL}}{\left(e^{mL} + e^{-mL}\right)} \quad ; \quad C_2 = \theta_s \frac{e^{-mL}}{\left(e^{mL} + e^{-mL}\right)}$$

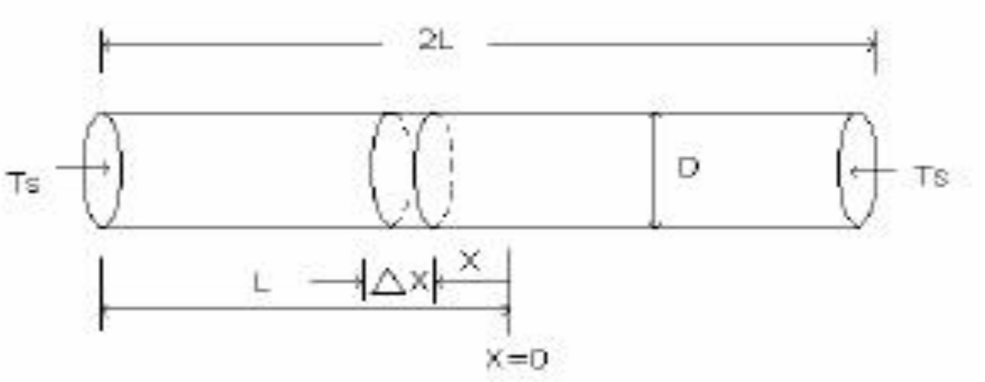


$$\frac{\theta}{\theta_s} = \frac{e^{m(L-x)} + e^{-m(L-x)}}{e^{mL} + e^{-mL}} \qquad \frac{\theta}{\theta_s} = \frac{\cosh m(L-x)}{\cosh mL}$$

$$Bi = \frac{h\ell'}{k} \qquad \ell' = \frac{PL^2}{A} \qquad Q = k.A.m.(Tw - T_\infty).\tanh mL$$

UNSTEADY STATE HEAT TRANSFER FROM HORIZANTAL ROD WHOSE ENDS ARE KEPT AT CONSTANT TEMPERATE

$$A.q\big|_{x} - A.q\big|_{x+\Delta x} - h.(T - T_{\infty})P.\Delta x = \rho.C_p.A.\Delta x.\frac{dT}{dt}$$



$$A\left(-k\frac{dT}{dx}\right)\Bigg|_{x} - A\left(-k\frac{dT}{dx}\right)\Bigg|_{x+\Delta x} - h.(T - T_{\infty}).P.\Delta x = \rho.C_p.A.\Delta x.\frac{dT}{dt}$$

$$\frac{\partial^2 T}{\partial x^2} - \frac{h.P}{k.A}(T - T_{\infty}) = \frac{\rho.C_p.A}{k.A} \times \frac{\partial T}{\partial t} \qquad \frac{\partial Q}{\partial t} = \frac{\partial T}{\partial t} \Rightarrow \frac{d^2 Q}{dt^2} = \frac{d^2 T}{dt^2}$$

$$t = 0 \quad T = T_{\infty} \qquad U = (T_i - T_{\infty})$$

$$x = L \quad T = T_s \qquad U = (T_s - T_{\infty})e^{\beta t} = Q_s e^{\beta t}$$

$$x = -L \quad T = T_s \qquad U = (T_s - T_{\infty})e^{-\beta t}$$

$$Q = Q_s \times \frac{\cosh x\left(\frac{\beta}{\alpha}\right)^{1/2}}{\cos Lx\left(\frac{\beta}{\alpha}\right)^{1/2}} - \frac{4Q_s}{\pi} \times \sum_{n=0}^{\infty} \frac{(-1)^n e^{-\beta t} - \left[\frac{\alpha(2n+1)^2 \pi^2 t}{4L^2}\right] \times \cos\left[\frac{(2n+1)\pi x}{2L}\right]}{(2n+1)\left[1 + \frac{4.\beta.L^2}{(2n+1)^2 \pi^2 \alpha}\right]}$$

$$Q = Q_s \frac{\cosh x\left(\frac{\beta}{\alpha}\right)^{1/2}}{\cosh L\left(\frac{\beta}{\alpha}\right)^{1/2}} - 0$$

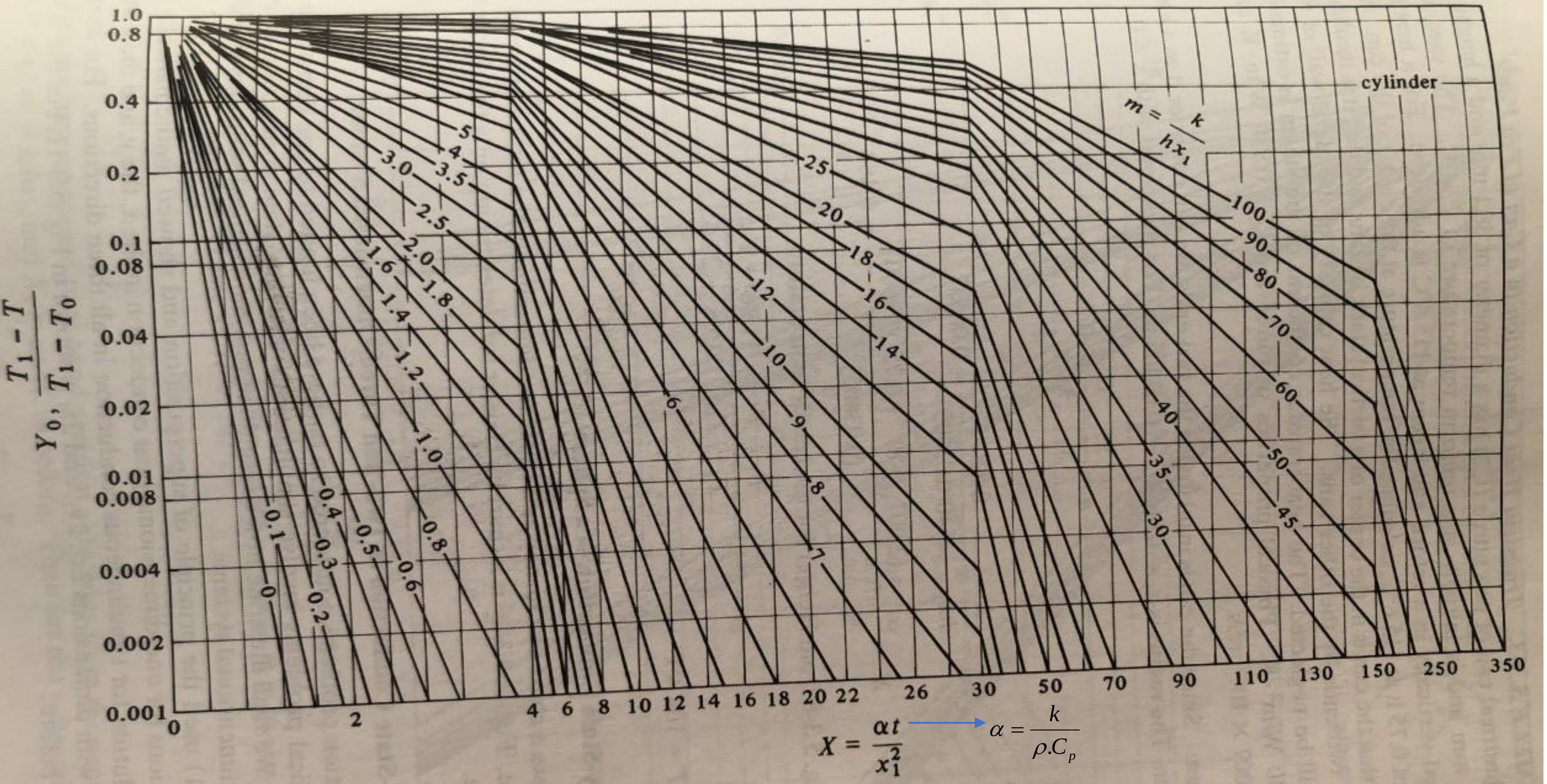
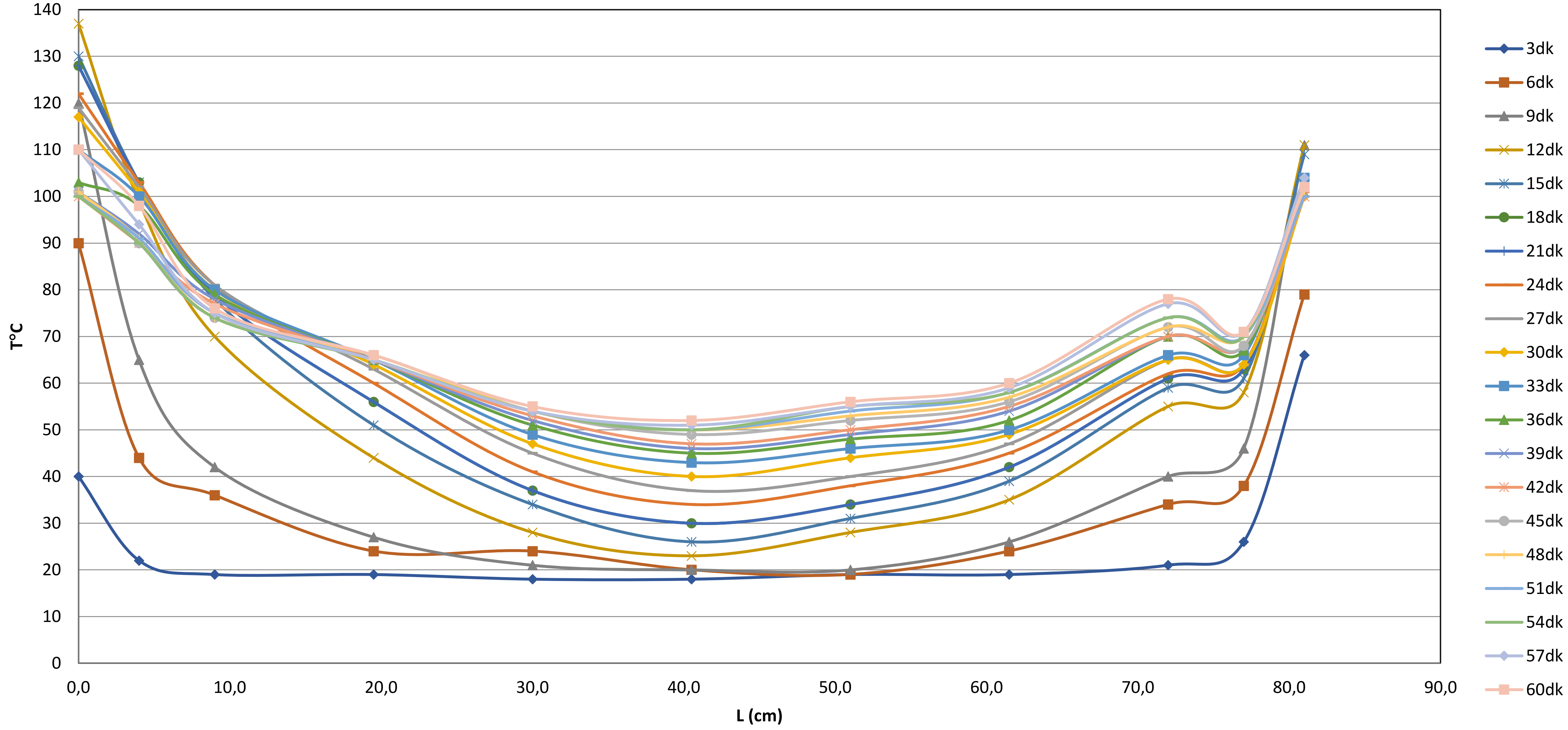


FIGURE 5.3-8. *Chart for determining temperature at the center of a long cylinder for unsteady-state heat conduction. [From H. P. Heisler, Trans. A.S.M.E.,* **69,** *227 (1947). With permission.]*


T°C
140
130
120
110
100
90
80
70
60
50
40
30
20
10
0
0,0
10,0
20,0
30,0
40,0
50,0
60,0
70,0
80,0
90,0
L (cm)
3dk
6dk
9dk
12dk
15dk
18dk
21dk
24dk
27dk
30dk
33dk
36dk
39dk
42dk
45dk
48dk
51dk
54dk
57dk
60dk

Data Table

Row Number	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9	T_{10}	T_{11}
1											
2											
3											
4											
5											
6											
7											
8											
9											
10											
11											
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