International Workshop on Dynamical Systems and Applications (IWDSA 2019)
In Memory of Prof. Dr. Aydın Tiryaki
Gazi University, Ankara, Turkey, 3-4 May 2019

The concept of Hukuhara derivative and Aumann integral for intuitionistic fuzzy number valued function

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Abstract

In this talk we will firstly define a new metric in intuitionistic fuzzy environment and study its properties. Then we will show that the metric space of fuzzy number valued functions is complete under this metric. Lastly, we will study the concept of the Aumann integration for intuitionistic fuzzy number valued functions in terms of α and β cuts. And we will give the proof of the relation between Hukuhara derivative and Aumann integral for intuitionistic fuzzy valued functions by using the fundamental theorem of calculus.

In science and technology, vagueness or ambiguity is an inevitable phenomena. Hence to understand and interpret the models containing elements of uncertainty, probabilistic or possibilistic approaches are developed. Generally, the possibilistic approaches are based on fuzzy set theory which was introduced by L. A. Zadeh in 1965 [1].

In 1986, Atanassov introduced the concept of intuitionistic fuzzy sets (IFS) and carried out rigorous researches to develop the theory [2]. In this set concept, he introduced a new degree $\nu : X \to [0, 1]$, called non-membership function, such that the sum $\mu + \nu$ is less than or equal to 1. Hence the difference $1 - (\mu + \nu)$ is regarded as degree of hesitation. In applications of fuzzy sets or intuitionistic fuzzy numbers generally α or β which are compact and convex sets are used. However, in those applications there is a problem : The inverse element of a compact and convex set with respect to Minkowski sum does not always exists. That is, if $A = \{a\}$ is not a singleton set, the Minkowski sum of A and $-A$ is not always the identity element $\{0\}$ i.e., $A + (-1)A \neq \{0\}$. Hence this is a drawback not only in theory of compact and convex sets but also in theory of fuzzy sets and IFS. That is why Hukuhara tried to handle the inverse element problem. He defined a new difference called Hukuhara difference (H-difference) for compact and convex sets [3]. Later Hukuhara difference of fuzzy sets and Hukuhara derivative (H-derivative) of fuzzy number valued functions were introduced and studied.

The concept of fuzzy integral was firstly defined by Sugeno [4]. Later Ralescu and Adams defined the fuzzy integral of positive measurable functions [5]. They studied some properties such as monotone convergence theorem and Fatou's lemma. Later Dubois and Parade generalized the Riemann integral over compact and convex sets to fuzzy valued functions [6]. This approach is mainly related with the concept of Aumann integral. Aumann integral is defined for set valued functions by Aumann [7]. Since a fuzzy-valued function is essentially a family of set-valued functions, fuzzy Aumann integration concept is employed in the concept of fuzzy integral and fuzzy differential equations.

Key Words: Intuitionistic fuzzy sets, Intuitionistic fuzzy number valued functions, Hukuhara derivative, Aumann integral, Intuitionistic Hausdorff metric.

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