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The spectra of the graph with using k-th power of a graph

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Abstract

A graph G consists of a set $V = \{v_1, v_2, v_3 ...\}$ called vertices and other set $E = \{e_1, e_2, e_3 ...\}$ whose elements are called edges. Usually the graph is denoted as $G = (V, E)$. A tree is an undirected graph in which any two vertices are connected by exactly one path. The path graph is a tree with two nodes of vertex degree 1, and the other nodes of vertex degree 2. A path graph is therefore a graph that can be drawn so that all of its vertices and edges lie on a single straight line. Star graph is a special type of graph in which $n-1$ vertices have degree 1 and a single vertex have degree $n-1$. The cycle C_n, $n \geq 3$, consists of n vertices $v_1, v_2, v_3 ..., v_n$ and edges $\{v_1, v_2\}$, $\{v_2, v_3\}$,...$\{v_{n-1}, v_n\}$ and $\{v_n, v_1\}$. The complement or inverse of a graph G is a graph $\overline{G}$ on the same vertices such that two distinct vertices of $\overline{G}$ are adjacent if and only if they are not adjacent in G. The $k-th$ power G^k of a graph G is a graph with the same vertex V such that two vertices are adjacent in G^k iff their distance in G is at most k. The adjacency matrix of G^k is denoted by A^k. The adjacency matrix of G^k is defined by its entries $a_{ij} = 1$ if $i \sim_k j$ and 0 otherwise. The eigenvalues of A^k are denoted by $\lambda_1^{(k)} \geq \lambda_2^{(k)} \geq ... \geq \lambda_{n-1}^{(k)} \geq \lambda_n^{(k)}$.

In this talk,we give new theorem for the spectra of a graph, using $k-$th power of a graph . For any graph G of order n,

$$\begin{aligned} 0 &\leq \sqrt[k]{\lambda_1(G^k)} \leq ... \leq \sqrt[3]{\lambda_1(G^3)} \leq \sqrt[2]{\lambda_1(G^2)} \\ &\leq \lambda_1(G) \leq \sqrt[k]{\lambda_1(G^k)} + \sqrt[k]{\lambda_1(\overline{G}^k)} \leq ... \\ &\leq \sqrt[3]{\lambda_1(G^3)} + \sqrt[3]{\lambda_1(\overline{G}^3)} \leq \sqrt[2]{\lambda_1(G^2)} + \sqrt[2]{\lambda_1(\overline{G}^2)} \end{aligned}$$

shaped is a decreasing sequence.

Key Words: k-th power of a graph, Spectra of the graph.

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