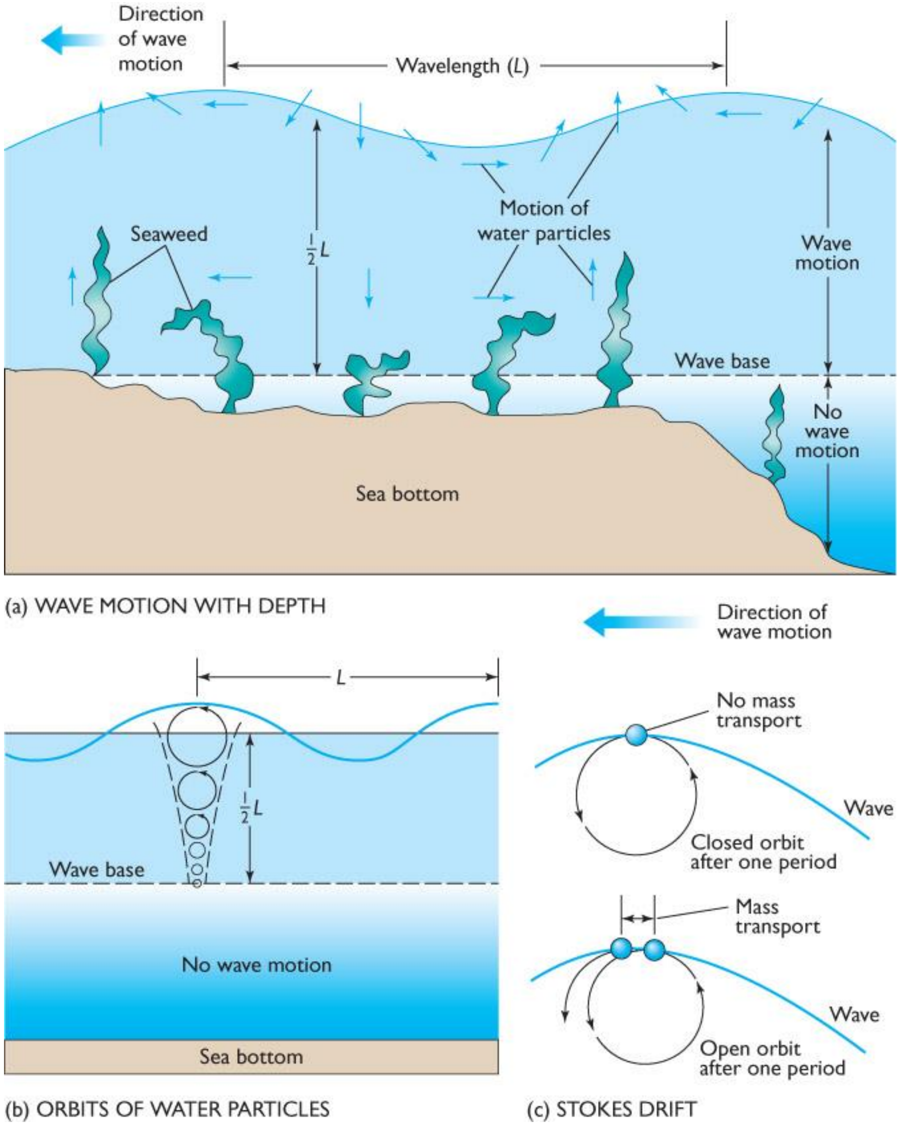
SUMMARY: The Motion of Water Particles Beneath Waves

Wave Motion with Depth

Orbits of Water Particles

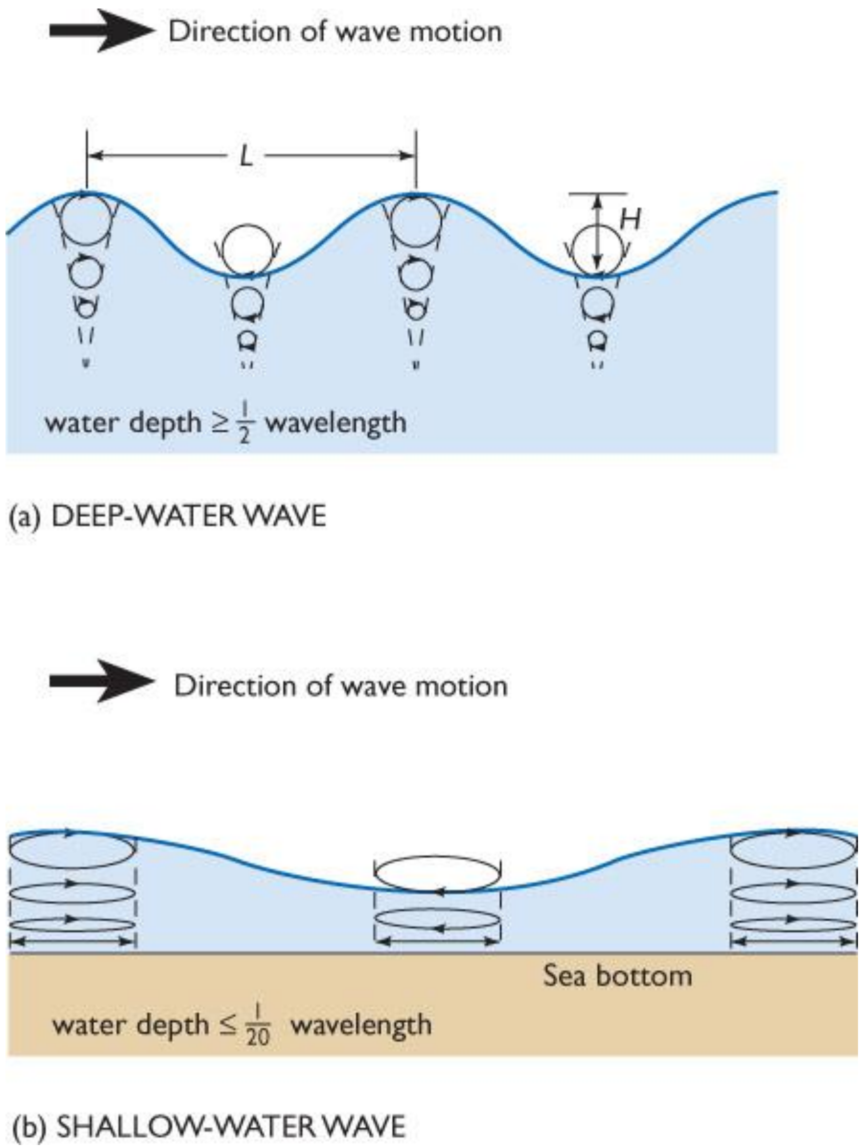


- Wave base is the depth to which a surface wave can move water.

- If the water is **deeper** than wave base:
  - orbits are circular
  - there is no interaction between the bottom and the wave

- If the water is **shallower** than wave base:
  - orbits are elliptical
  - Orbits become increasingly flattened towards the bottom

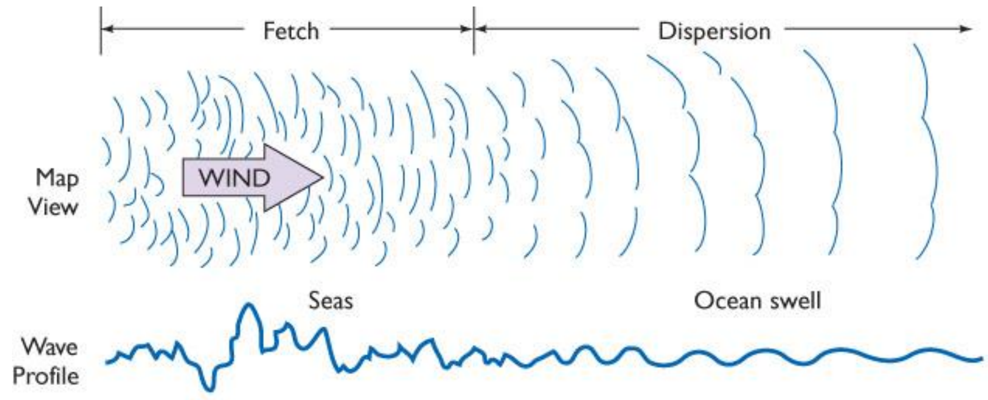
The Distortion of Water-Particle Orbits in Shallow Water



- **Celerity** is the velocity of the **wave form,** not of the water, C=L/T

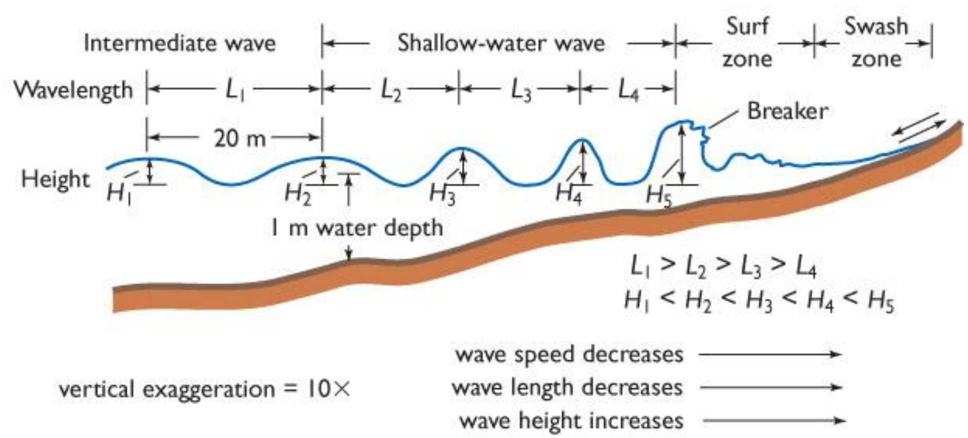
- The celerity of a group of waves all traveling at the same speed in the same direction is **less than** the speed of individual waves within the group, Cg=nC

Wave Transformations



(a) DEEP-WATER WAVE TRANSFORMATIONS

Deep-Water Wave Transformations



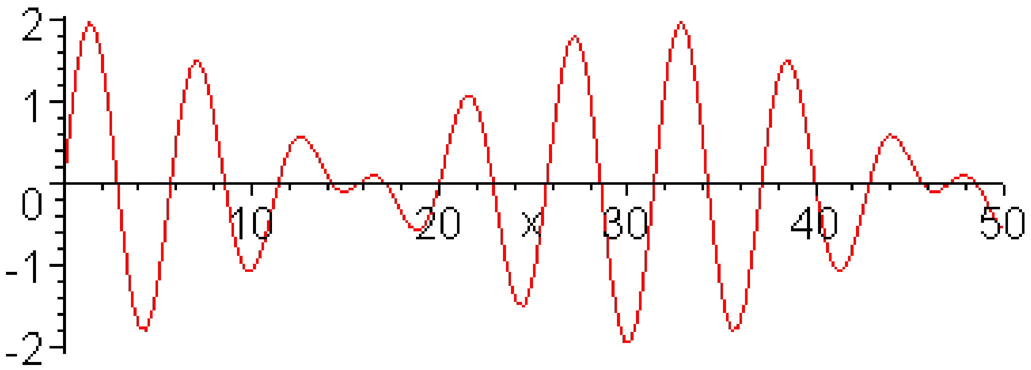
(b) SHALLOW-WATER WAVES IN PROFILE

Shallow-Water Waves in Profile

- Dispersion refers to the sorting of waves of different sizes with time. If wave speeds are dependent on the wavenumber (e.g., deep-water Airy waves), the wave-field is said to be **dispersive**.
- If the wave speed is independent of wavenumber (e.g., shallow-water Airy waves), the wave-field is **non-dispersive**.

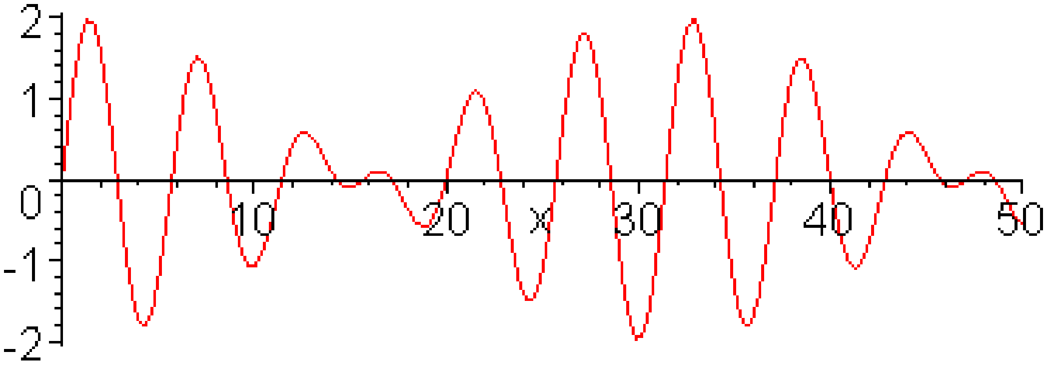
- **Dispersive sinusoidal waves**
- **In normal dispersion, c increases with L, ie the crests of long waves travel faster than the crests of short waves. As a consequence cg < c, ie energy travels slower than the wave crests. This occurs with gravity waves in intermediate water depths**

  Deep water wave speed
  (short waves; depth is larger than 1/2 of the wavelength) cg = c / 2 (normal dispersion)



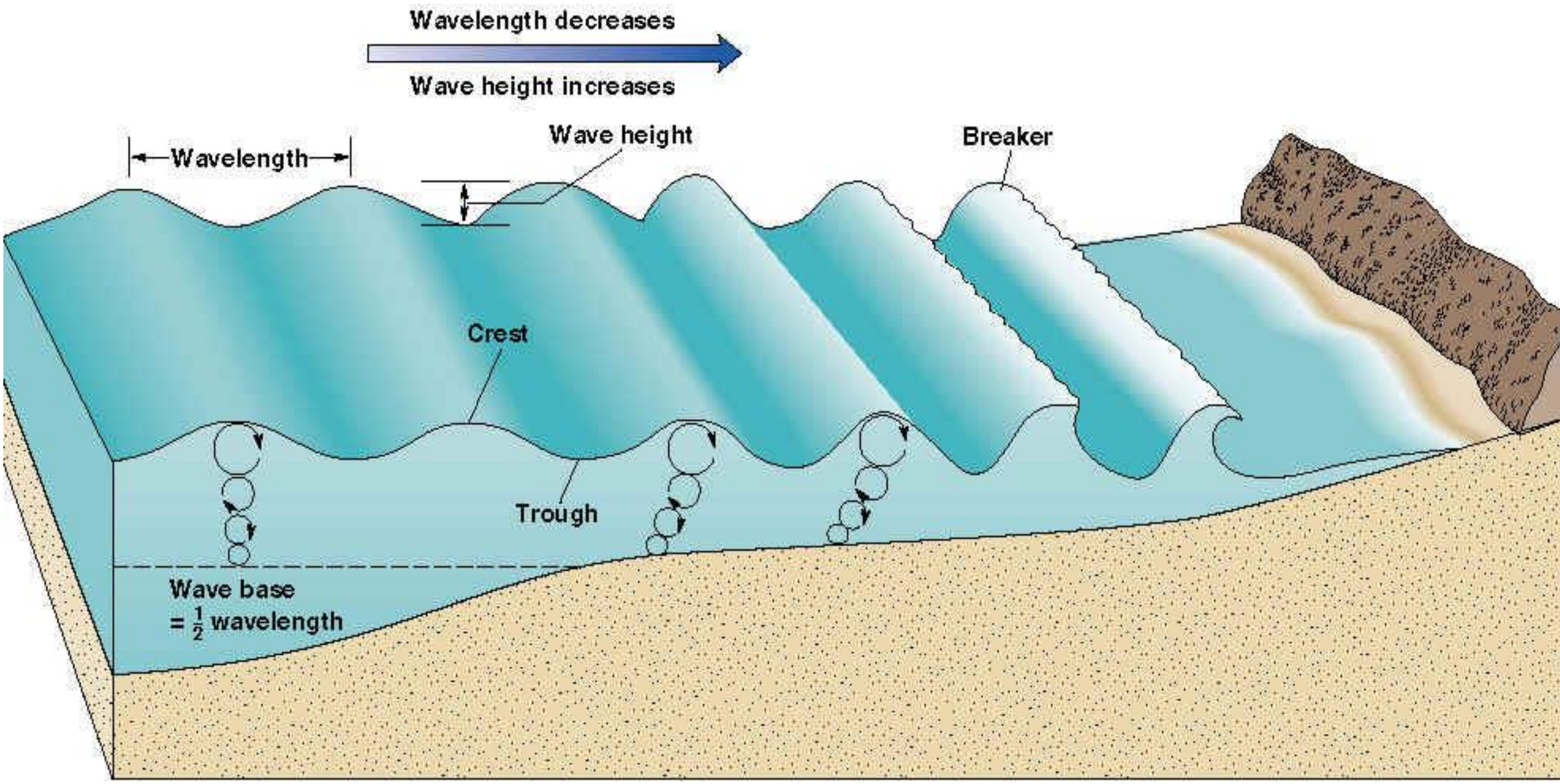
Nondispersive waves

Here, cg = c, ie all wave crests travel at the same speed, and energy propagates at the same speed. Shallow water wave speed (long waves; depth is less than 1/20 of the wavelength) cg = c (nondispersive waves)



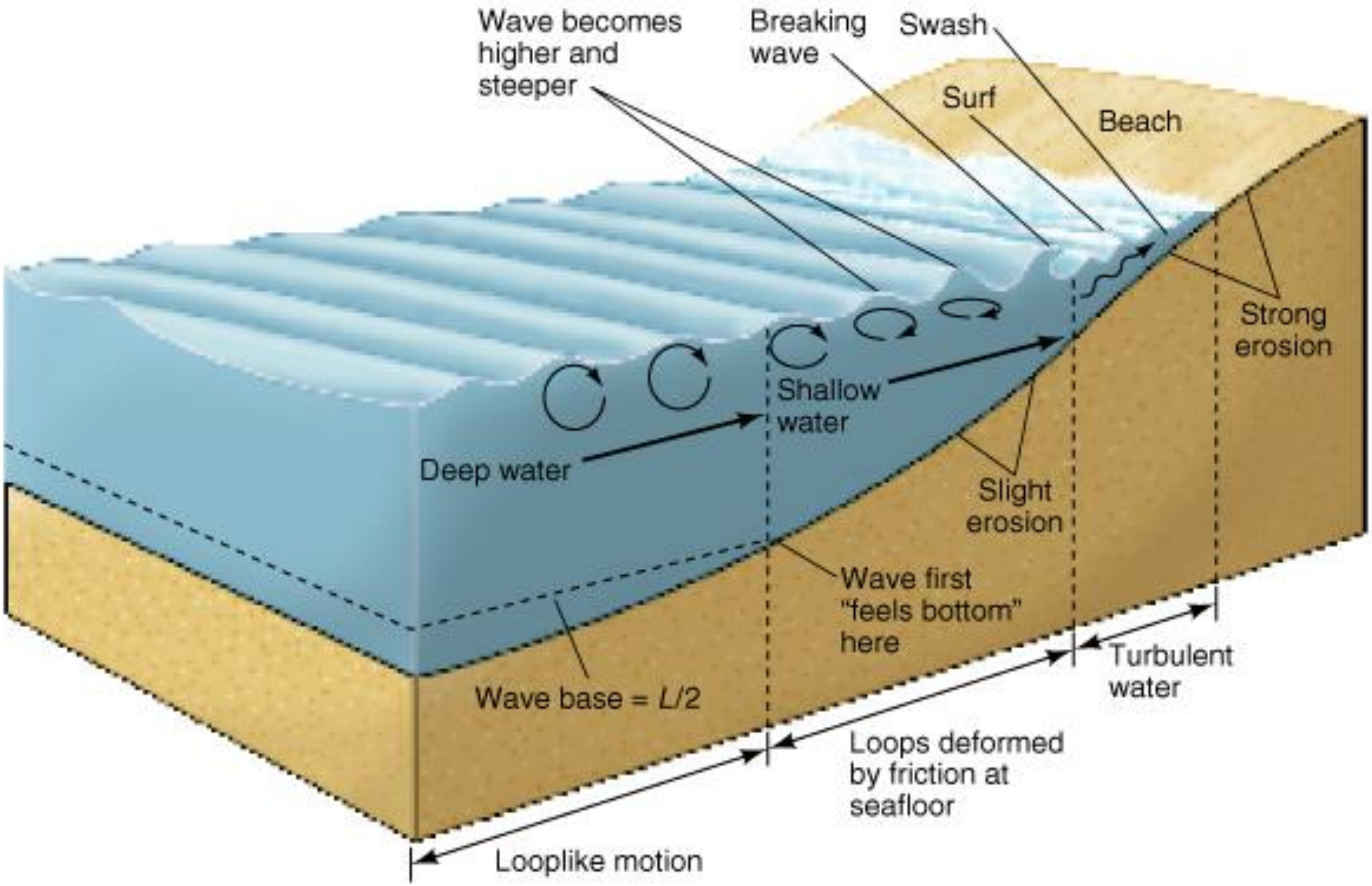
Wave steepness is a ratio of **wave height** divided by **wavelength** (H/L)

- wave height increases, as depth decreases
- wave length decreases, as depth decreases
- When $H/L \geq 1/7$, the wave becomes unstable and **breaks**.



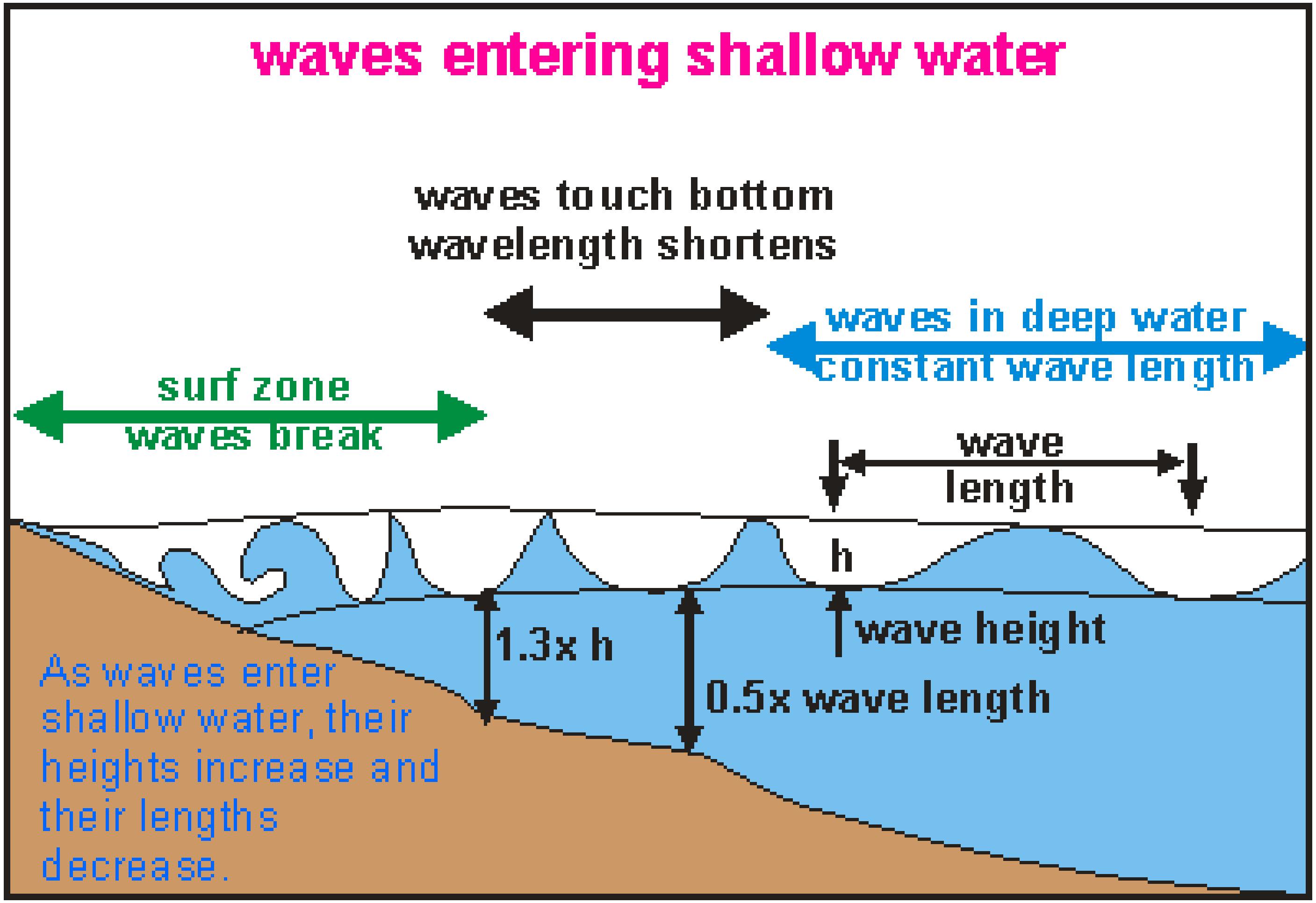
Wavelength decreases
Wave height increases
Wave height
Breaker
Wavelength
Crest
Trough
Wave base
= ½ wavelength

Wave Transformation



Wave Transformation

Waves break on the shore because they run out of room and all of their energy crashes in.



WAVE TRANSFORMATIONS

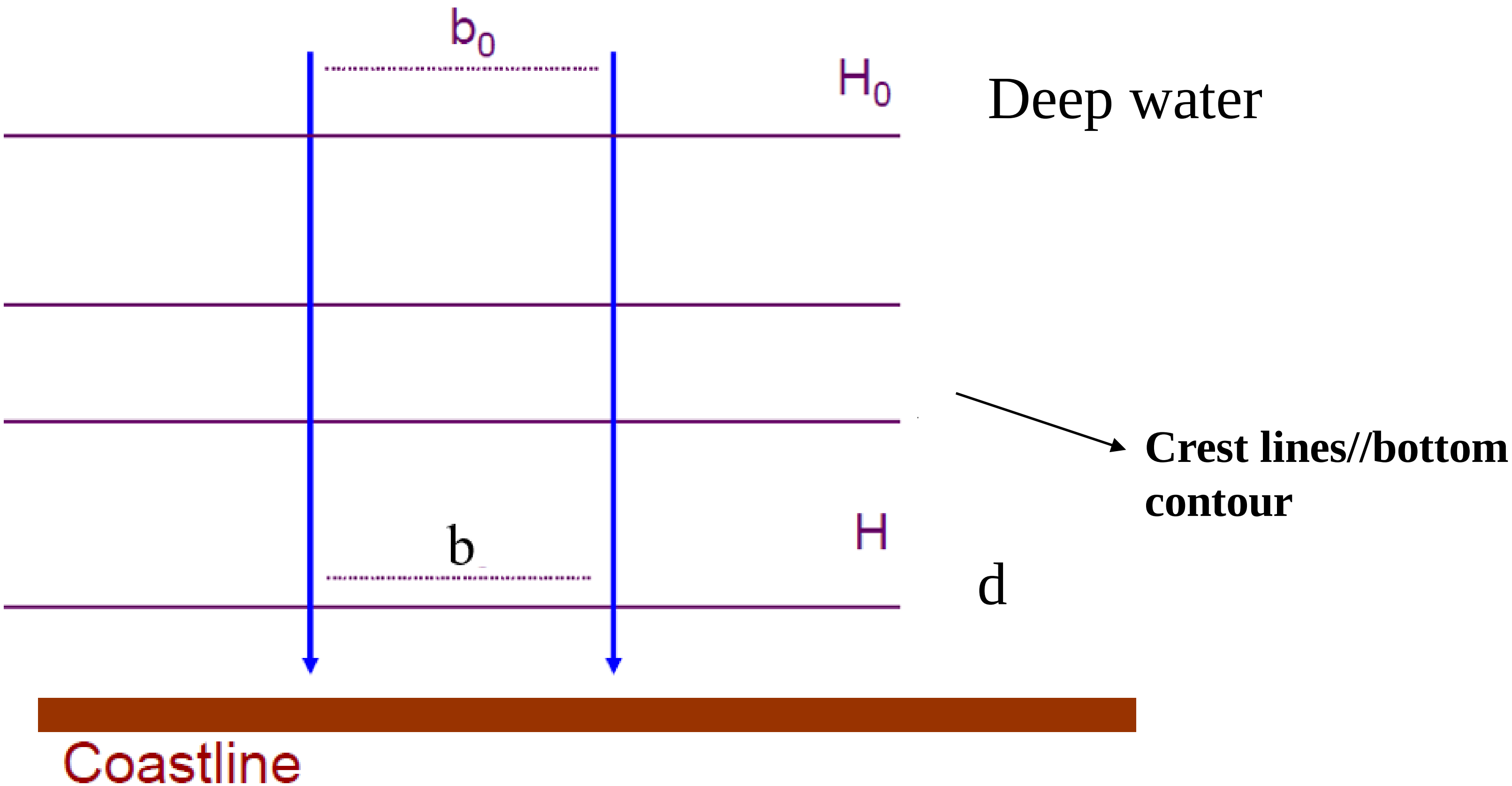
WAVE TRANSFORMATIONS

Wave transformation is concerned with the changes in H, L, C and α, the wave angle with the bottom contours; wave period T remains constant throughout the process.

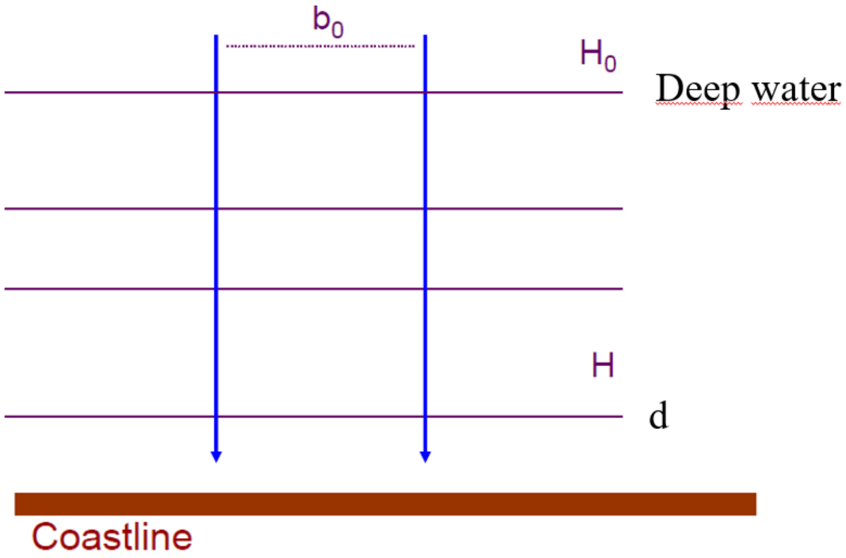
- Shoaling
- Refraction
- Diffraction
- Reflection

WAVE SHOALING

Consider only effect of shoaling, so that waves are approaching in a perpendicular direction to shoreline from deep water. B=Bo=constant



We can use our energy theory to apply to waves shoaling – moving into gradually decreasing water depths. Initially we consider the situation where a regular wave train moves so that the wave crests are parallel to the depth contours. This implies that here the energy contained within a certain width in deep water remains within the same width in shallow water.



- **Bottom contours are parallel to crest lines**

wave behaves as if it were in locally constant depth. Hence, the power crossing any section at one point is the same as anywhere else, and so

$$P = E\,c_g = E_0\,c_{g0},$$

where the 0 subscripts denote the deep water values. Thus we have

$$\frac{1}{8}\rho g H^2 c_g = \frac{1}{8}\rho g H_0^2 c_{g0},$$

or,

$$\boxed{\frac{H}{H_0} = \sqrt{\frac{c_{g0}}{c_g}} = K_s,}$$

the Shoaling Coefficient, showing that as the local group velocity c_g decreases with water depth, the wave height increases. We can substitute our expressions to give

$$K_s = \sqrt{\frac{c_0}{c} \times \frac{1}{1 + \frac{2kd}{\sinh 2kd}}},$$

where c_0 is the wave speed in deep water and c that at depth d. We can relate these two by writing $c_0 = \sigma / k_0$ and $c = \sigma / k$, where σ is the frequency of the waves, which is the same whether they are in deep or shallow water, giving

$$\frac{c_0}{c} = \frac{k}{k_0}$$

and in fact we can use the linear dispersion relation for the frequency at both points:

$$\sigma = \sqrt{gk \tanh kd} = \sqrt{gk_0},$$

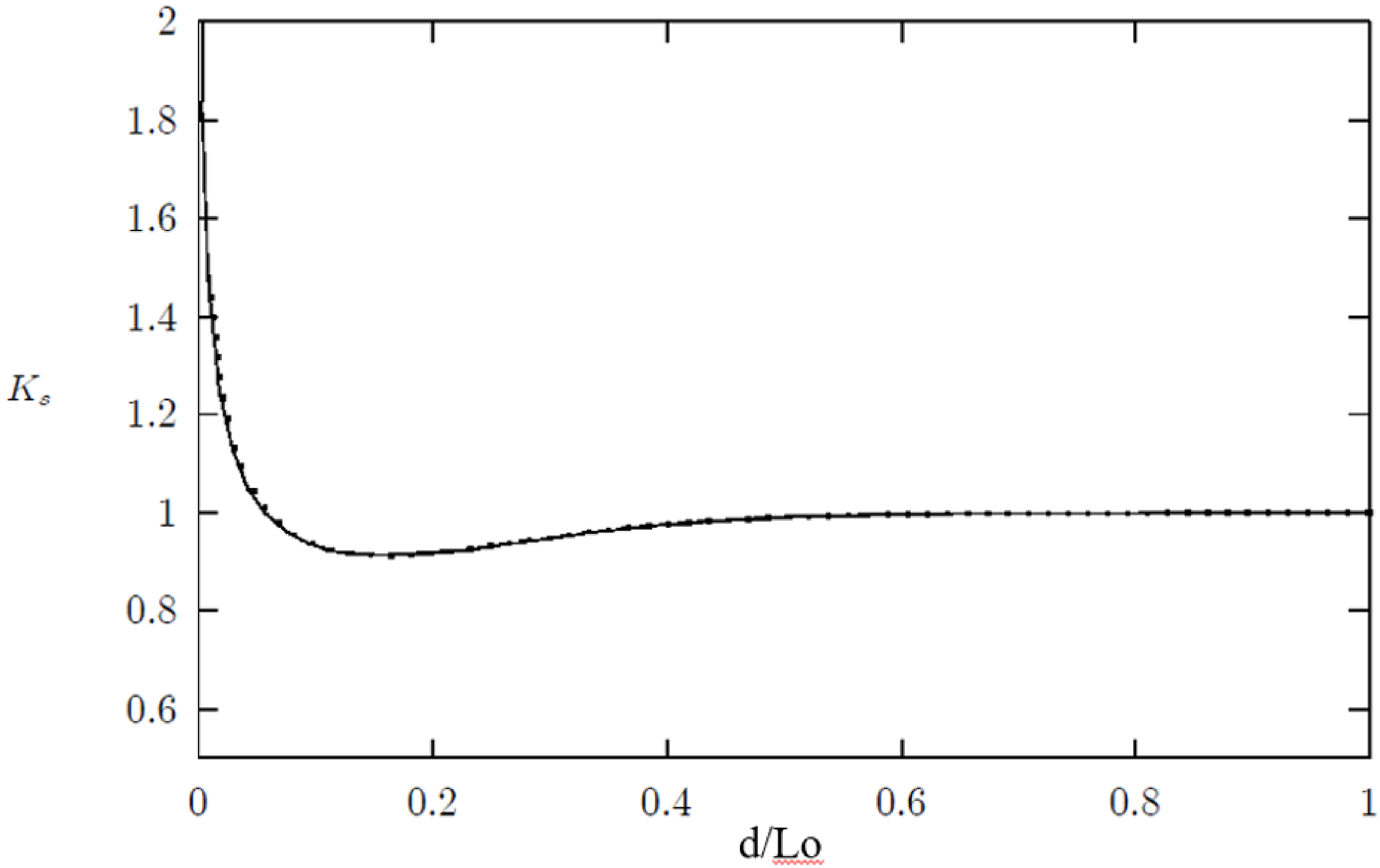
as in deep water, when $d \to \infty$, $\tanh kd \to 1$. This gives

$$\frac{k}{k_0} = \frac{1}{\tanh kd},$$

$$\begin{aligned} K_s &= \left(\tanh kd \left(1 + \frac{2kd}{\sinh 2kd}\right)\right)^{-1/2} \\ &= \left(\tanh kd + kd\left(1 - \tanh^2 kd\right)\right)^{-1/2}. \end{aligned}$$

$$\frac{H}{H_0} = \sqrt{\frac{c_{g0}}{c_g}} = K_s,$$

Last column of GWT
Given as function of
d/Lo or d/L



$$\frac{H}{H_0} = \sqrt{\frac{c_{g0}}{c_g}} = K_s,$$

Only for shoaling

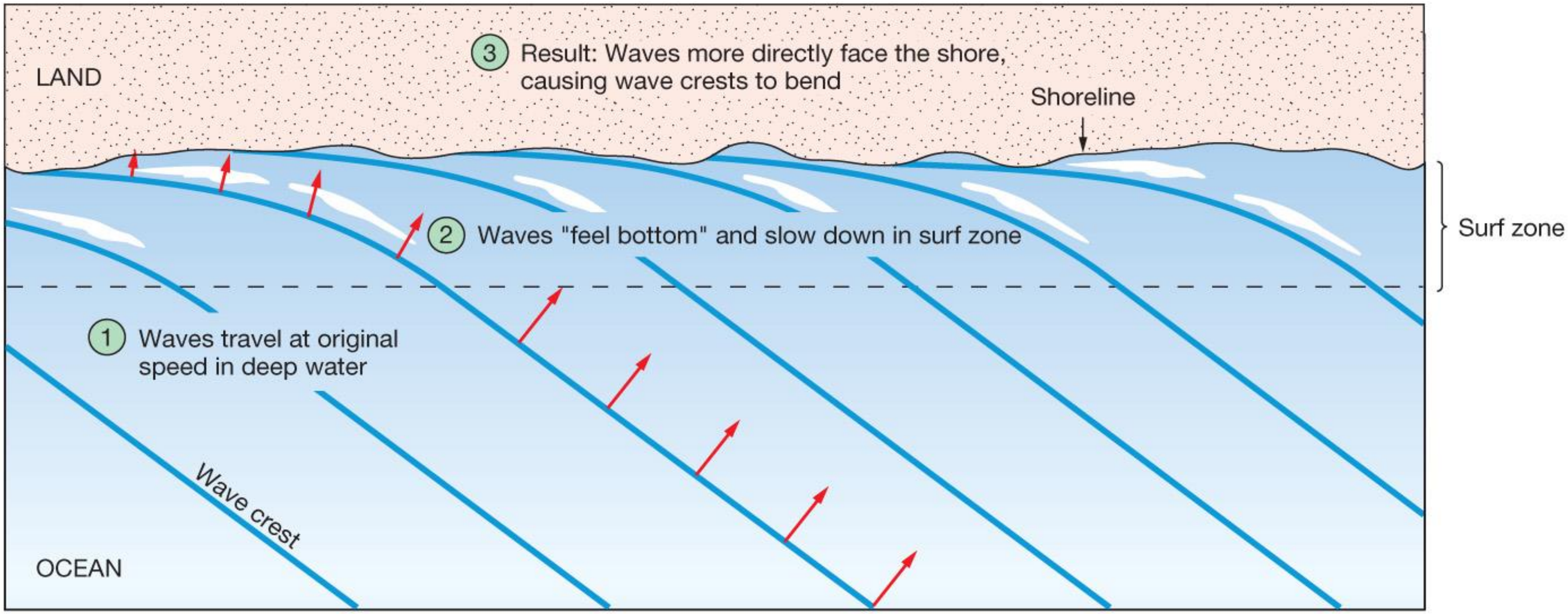
WAVE REFRACTION

Water waves travel faster on the surface of deep water than they do on shallow water. The change in speed of the wave will cause **refraction**.

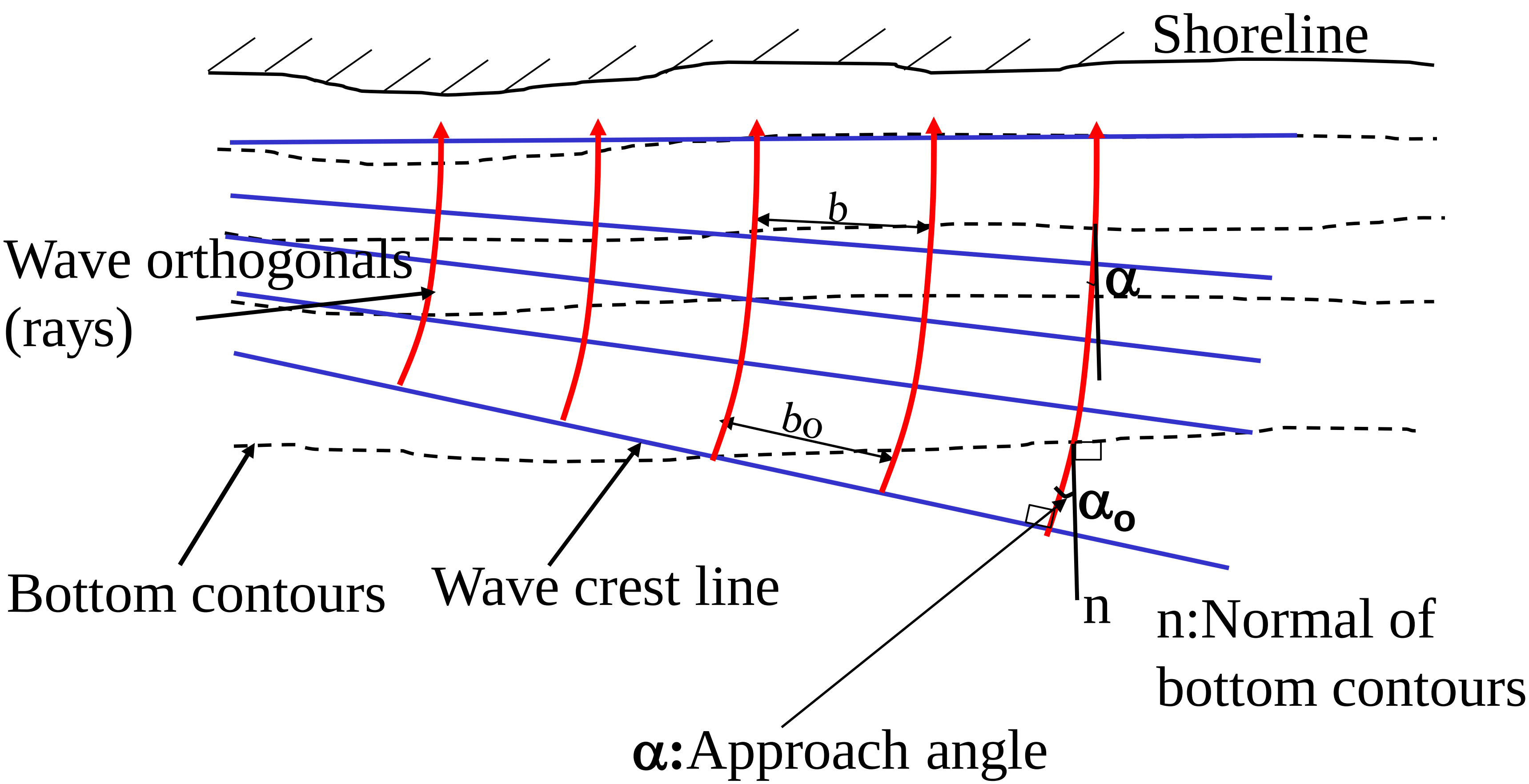
Wave Refraction

- Waves rarely approach shore at a perfect 90 degree angle.
- As waves approach shore, they bend so wave crests are nearly parallel to shore.
- Wave speed is proportional to the depth of water (shallow-water wave).
- Different segments of the wave crest travel at different speeds.

Wave Refraction



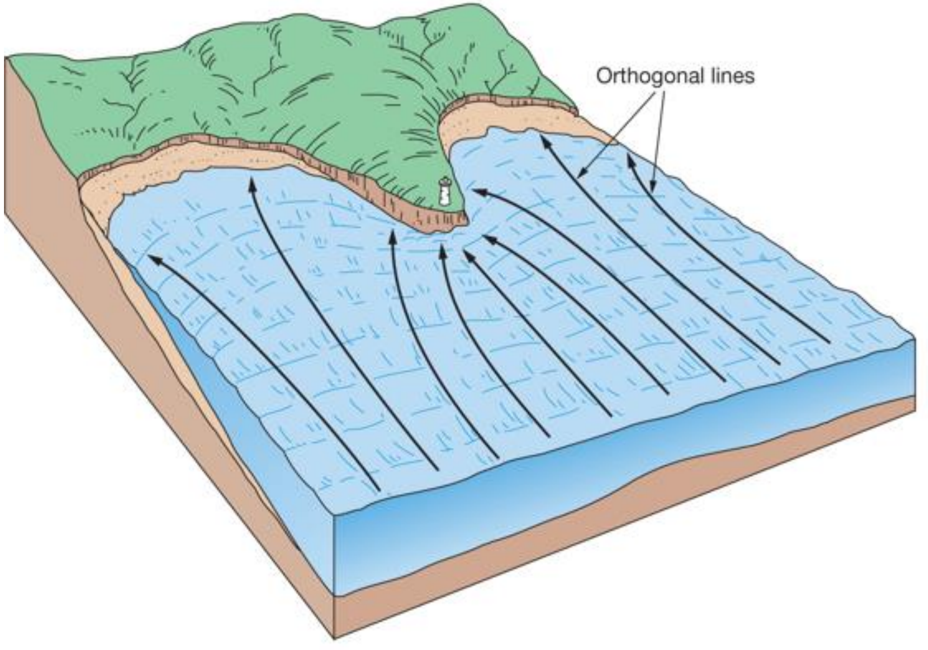
- Shoaling waves bend so wave fronts approach a shore nearly parallel



We can now draw wave rays (lines representing the direction of wave propagation) perpendicular to the wave crests and these wave rays bend

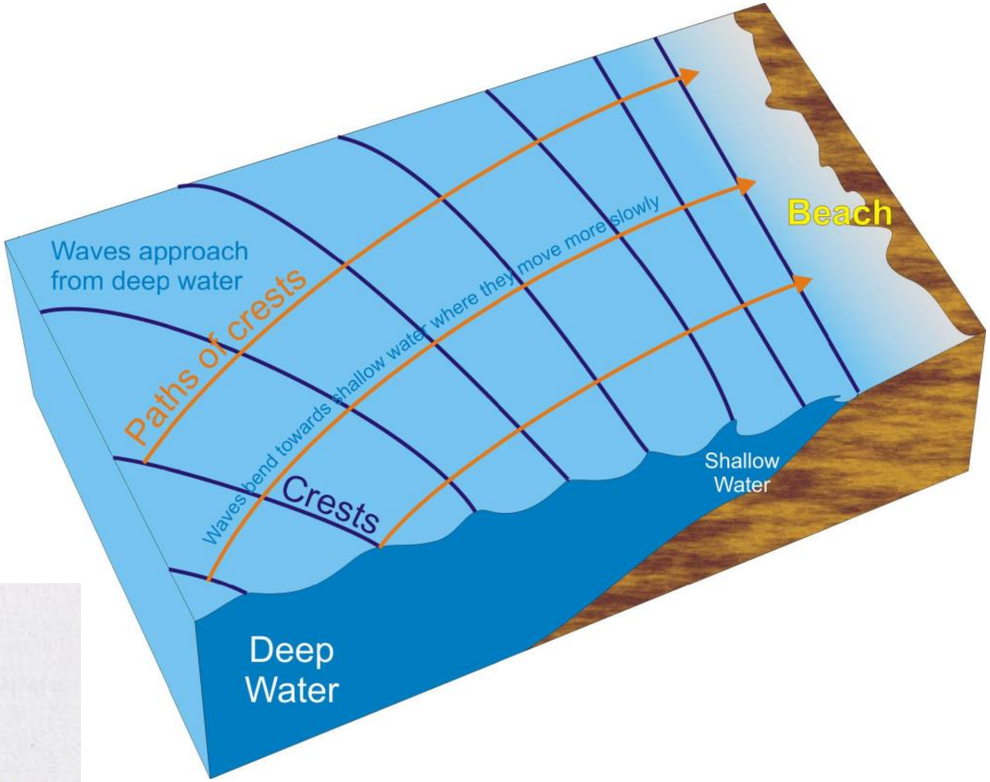
Wave Refraction

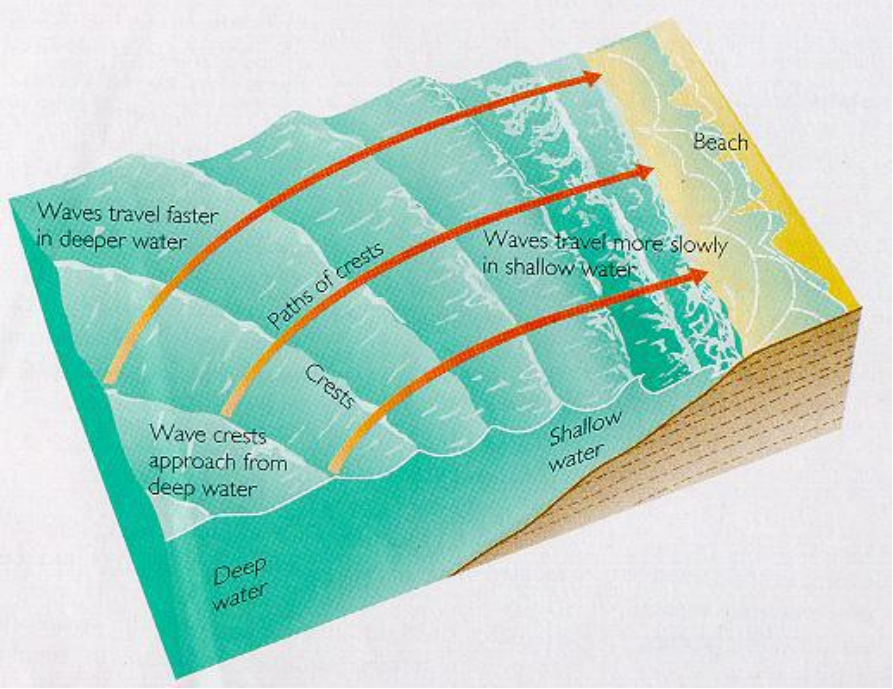
- Wave energy focused on headland
- Wave energy dispersed over bay



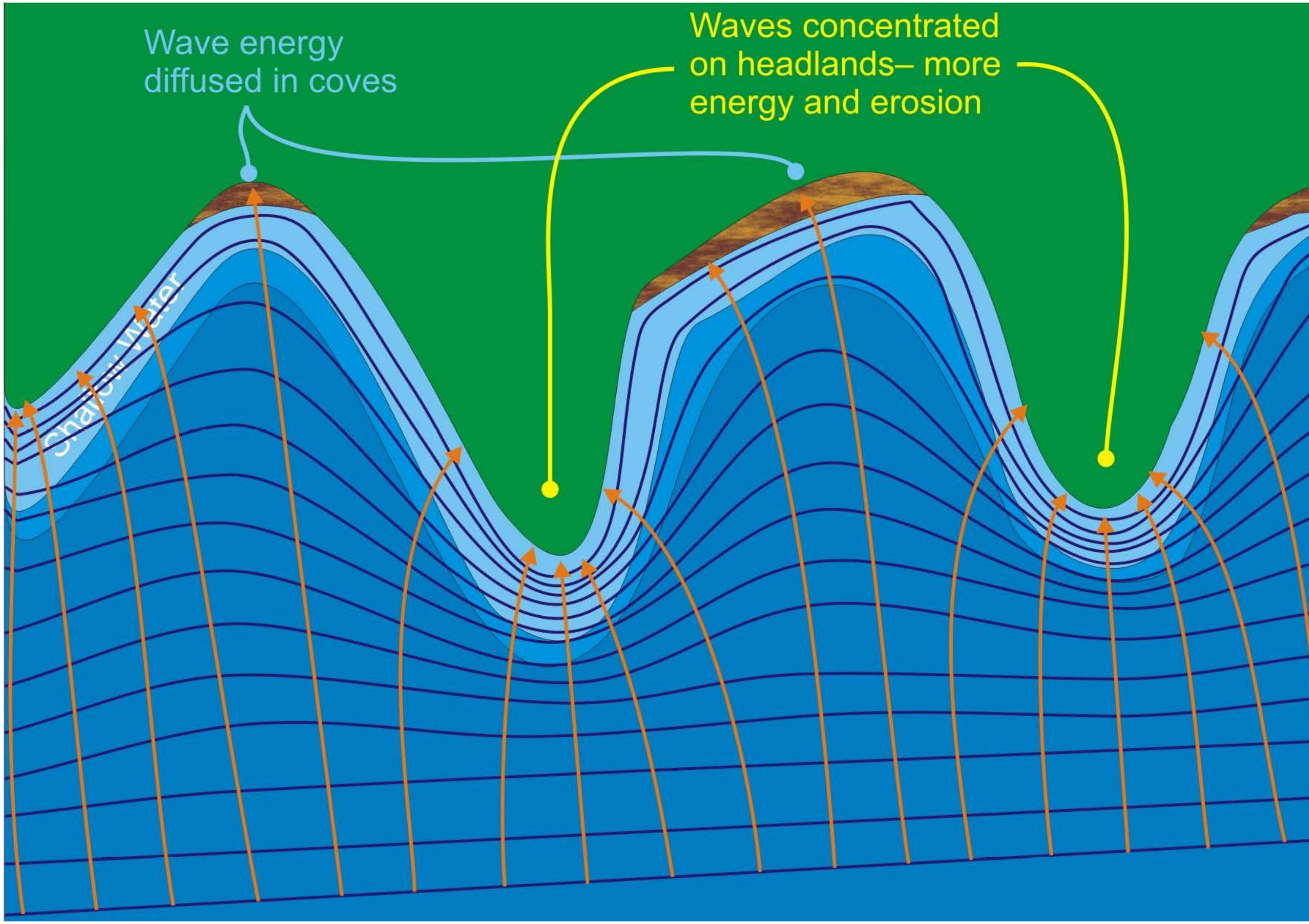
Wave Refraction

- Gradually erodes headlands
- Sediment accumulates in bays

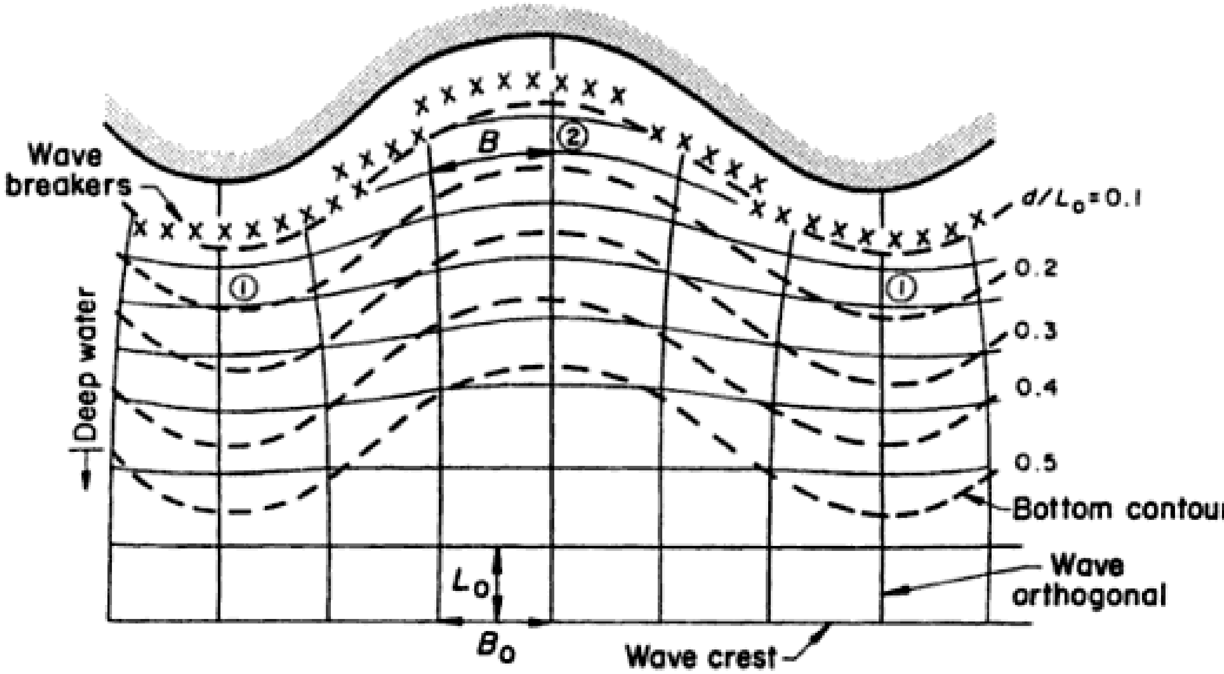




A cartoon showing how water waves refract towards shallow water.



A cartoon showing how water waves refract to erode headlands into arches and seastacks. As deep water waves approach the land, the shallow water around the headlands acts to concentrate the waves and their energy at the points, heightening erosion at the tips of the headlands. The energy of the waves that reaches the sandy coves is thus reduced, so that wave erosion isn’t so strong.



$$P_o = n_o C_o E_o b_o = \frac{1}{2} C_o (\frac{1}{8} \rho g H_o^2) b_o$$

The average energy flux

$$P = nC\,E\,b = nC\ (\frac{1}{8} \rho g H^2) b$$

$$P_o = P$$

$$\frac{1}{2} C_o (\frac{1}{8} \rho g H_o^2) b_o = nC\ (\frac{1}{8} \rho g H^2) b$$

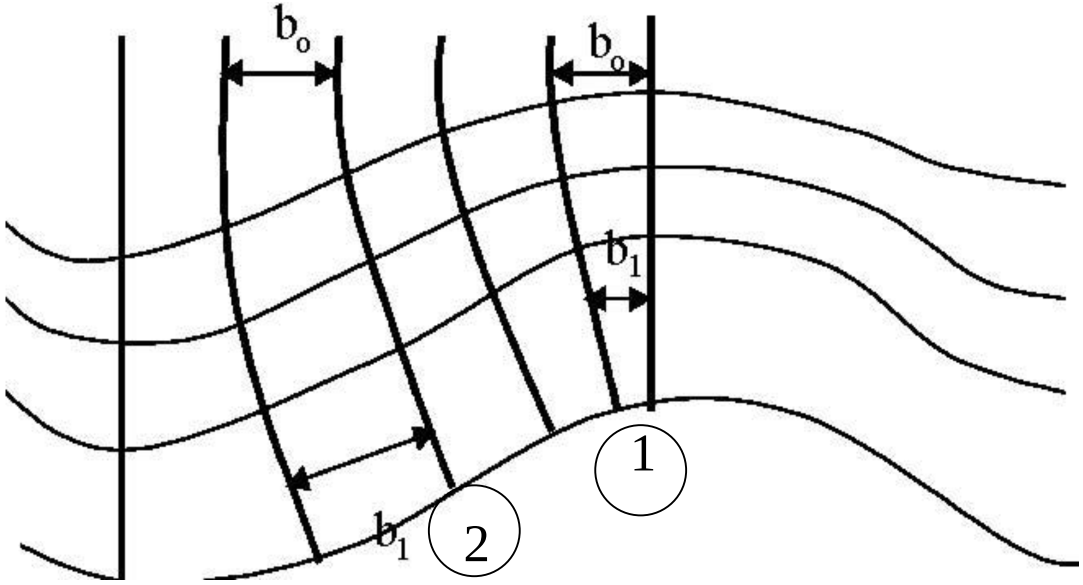
$$\frac{H}{H_o} = \sqrt{\frac{0.5 C_o}{Cn}} \sqrt{\frac{b_o}{b}}$$

Orthogonal spacing ratio $\frac{B_o}{B}$ must be determined from refraction analysis.

$$\frac{H}{H_o}=\sqrt{\frac{C_o n_o}{Cn}}\sqrt{\frac{B_o}{B}}$$

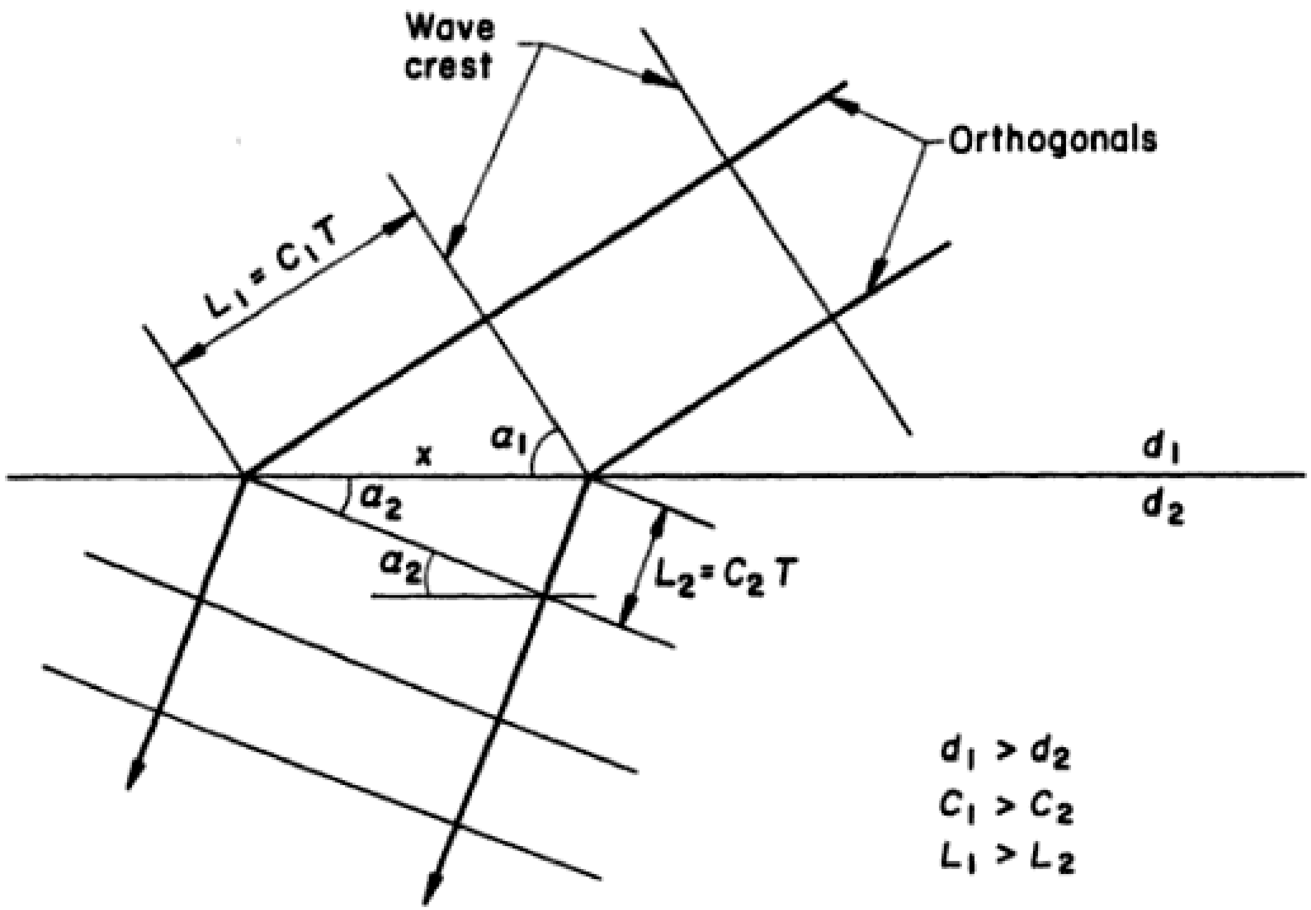
$$\frac{H}{H_o}=\sqrt{\frac{C_o 0.5}{Cn}}\sqrt{\frac{B_o}{B}}$$

$$\frac{H}{H_o}=K_s K_r$$



Refraction causes a convergence of orthogonals over the submerged ridge (point 1), resulting in higher waves that break.
Over the submerged trough (point 2), wave heights are lower than they would be over the ridge and can actually be much lower than the deep water wave height (if the effect of refraction in lowering the wave height is greater than the increase in wave height due to shoaling effect

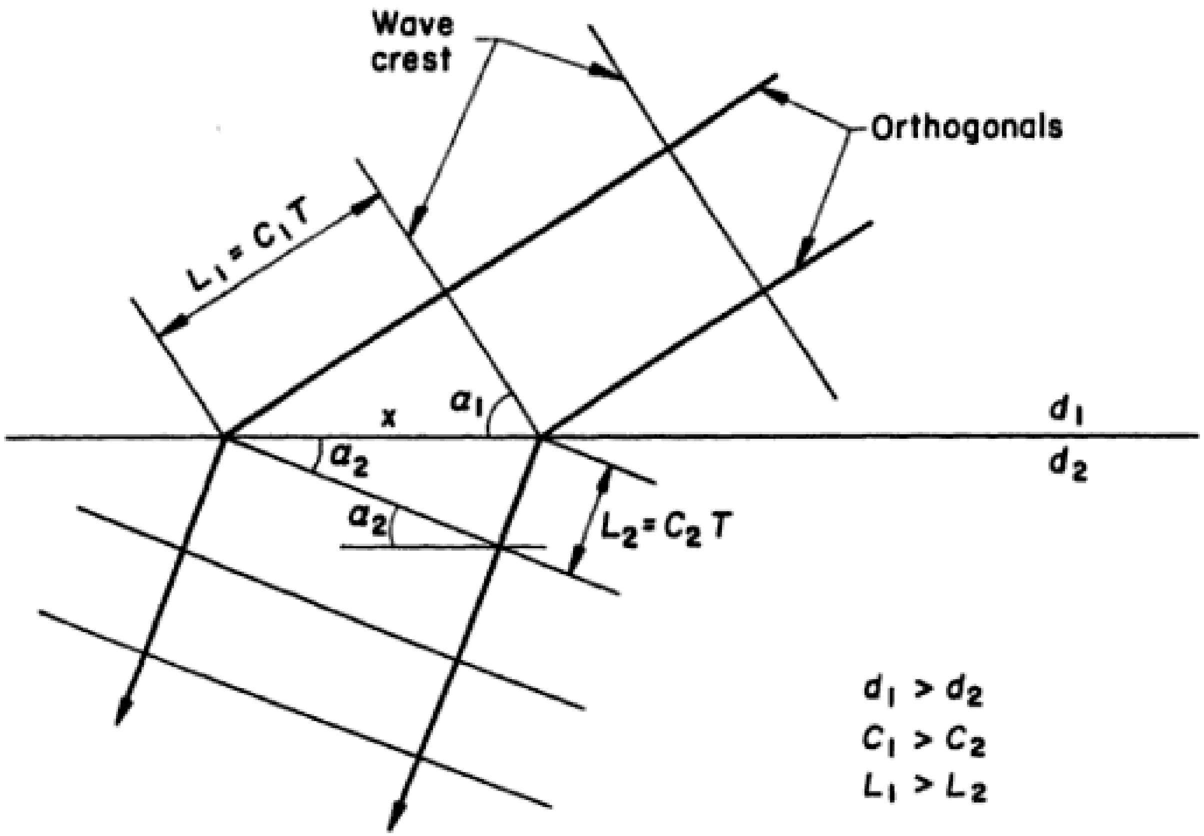
Wave approach angle α is the angle between the wave orthogonal and normal of the bottom contour or the angle between wave crest line and the bottom contour.



If in a wave train of waves propagating over a step where the water depth instantaneously decreases from d1 to d2, this causes the wave celerity and length to decrease. For an orthogonal spacing x and a time interval T,

$$\sin\alpha_1 = \frac{C_1 T}{x} = \frac{L_1}{x}$$

$$\sin\alpha_2 = \frac{C_2 T}{x} = \frac{L_2}{x}$$



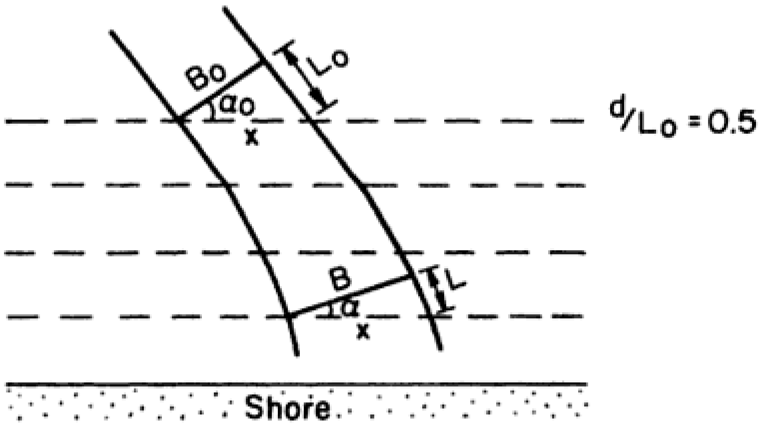
SNELL'S LAW

$$\frac{\sin\alpha_1}{\sin\alpha_2} = \frac{C_1}{C_2} = \frac{L_1}{L_2}$$

$$\frac{\sin\alpha_o}{L_o} = \frac{\sin\alpha}{L} = \frac{1}{x} \qquad \alpha = \sin^{-1}\left(\frac{L}{L_o}\sin\alpha_o\right)$$

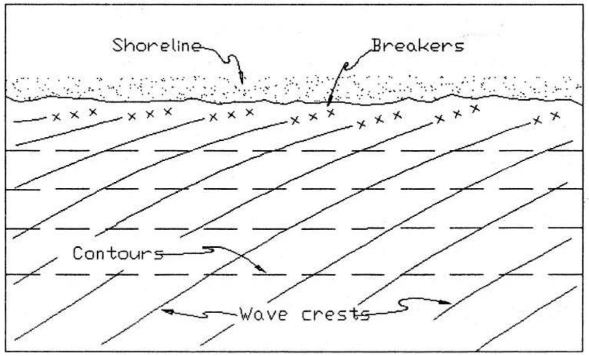
If we choose Bo and B so that the orthogonal lengths are equal to Lo and L, then

$$\frac{B_o}{\cos\alpha_o} = \frac{B}{\cos\alpha} = x$$

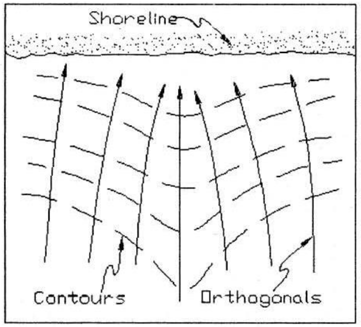


$$Kr = \sqrt{\frac{B_o}{B}} = \sqrt{\frac{\cos\alpha_o}{\cos\alpha}}$$

Refraction coefficient

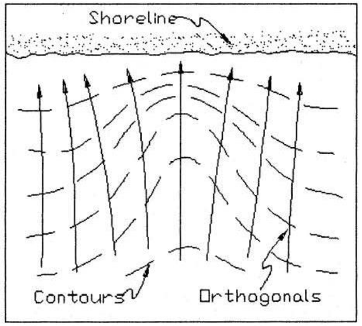


Shoreline
Breakers
Contours
Wave crests



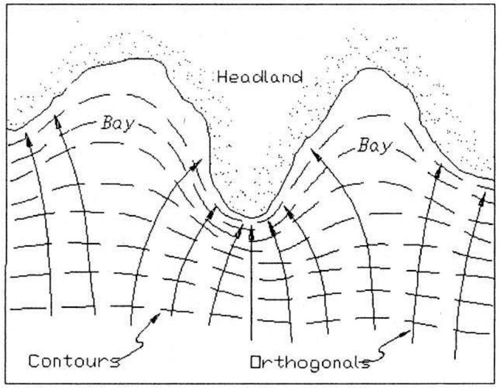
Shoreline
Contours
Orthogonals

(a)

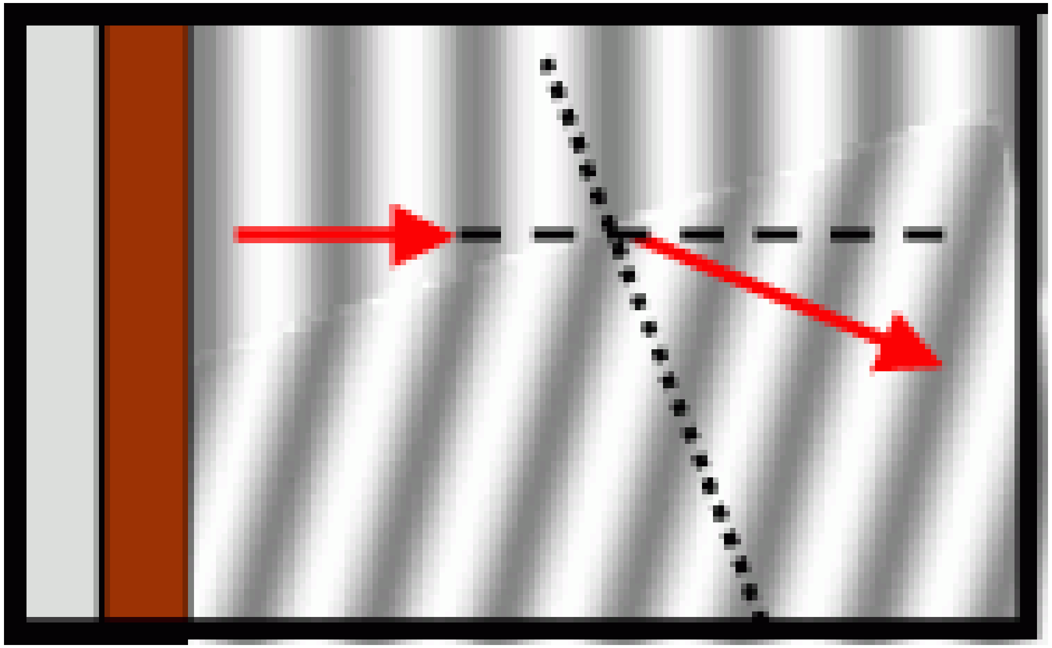


Shoreline
Contours
Orthogonals

(b)

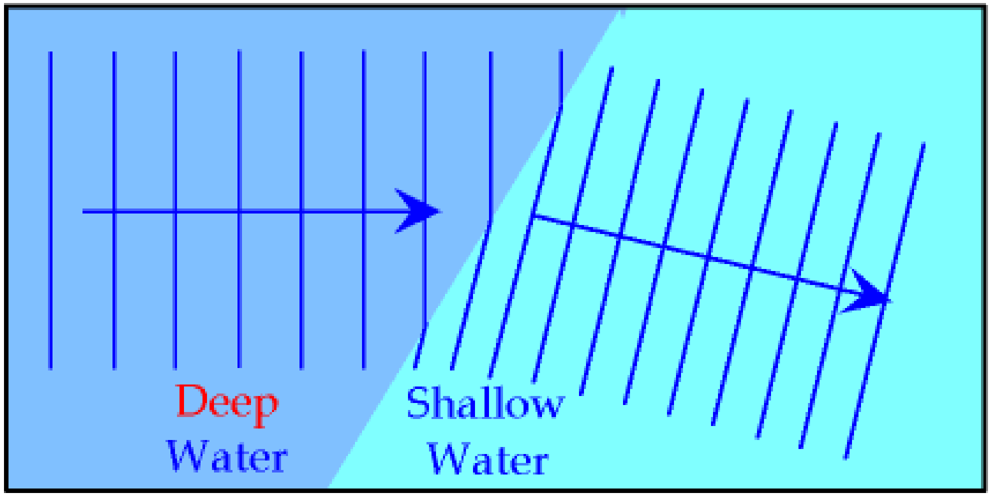


Headland
Bay
Bay
Contours
Orthogonals



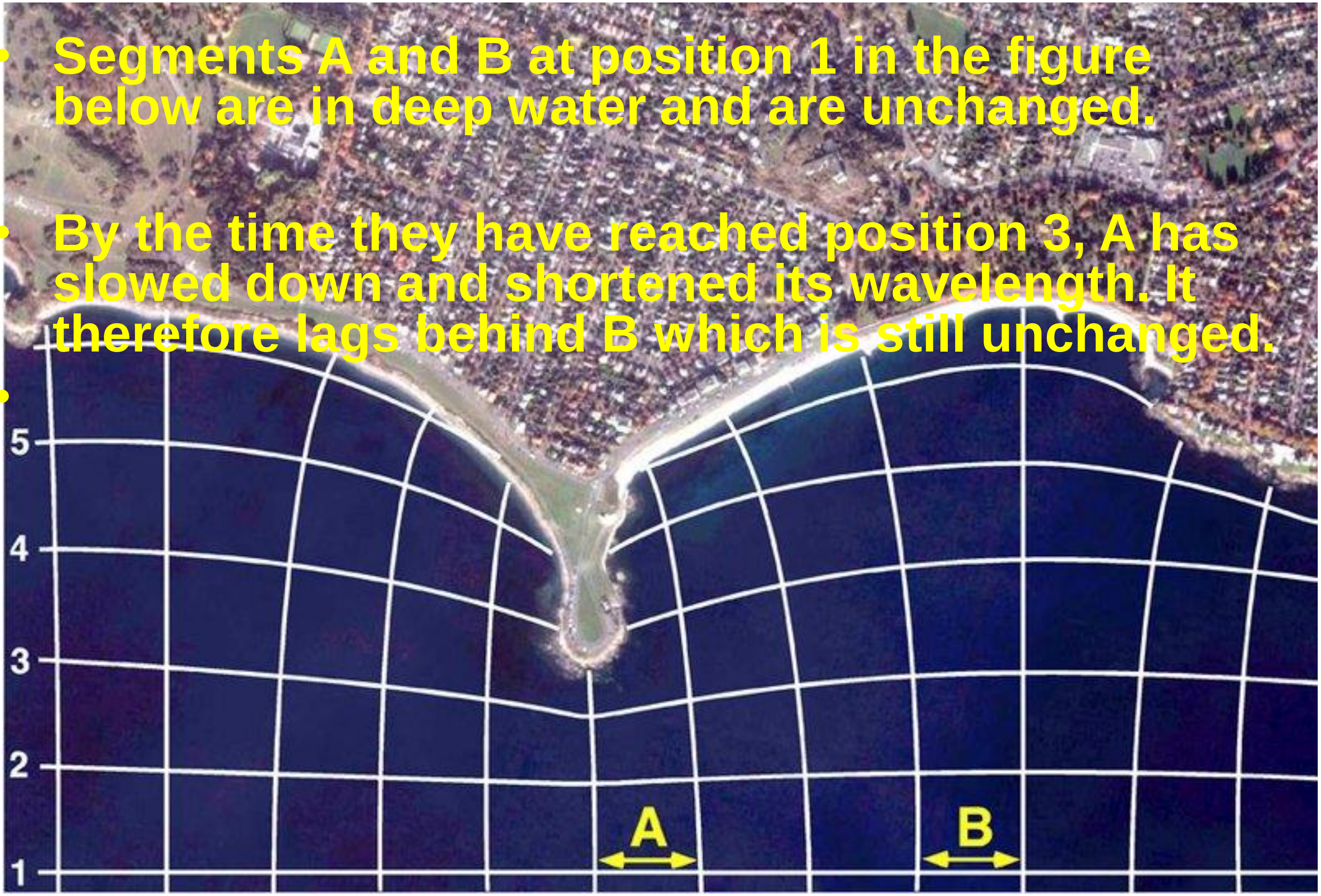
Water waves travel slower in shallower water.

Again the wavelength of the wave changes as the speed changes. The slower the speed of the wave the shorter its wavelength will be. This keeps the frequency the same.



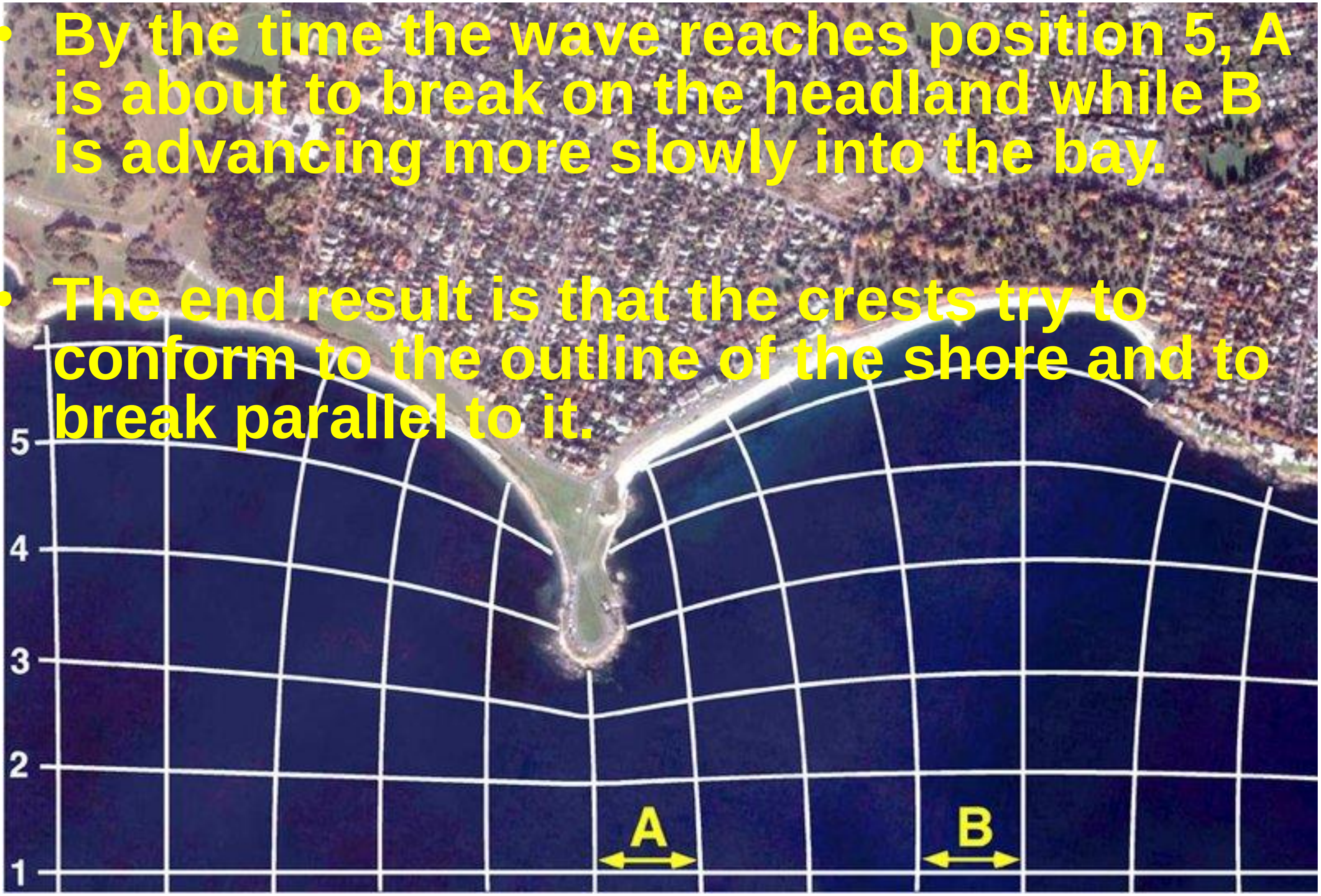
- **The change in speed has changed the direction of the wave.**
  The slower wave in the shallow water has a smaller wavelength
- **The amount of refraction increases as the change in speed increases**

- Segments A and B at position 1 in the figure below are in deep water and are unchanged.
- By the time they have reached position 3, A has slowed down and shortened its wavelength. It therefore lags behind B which is still unchanged.
-

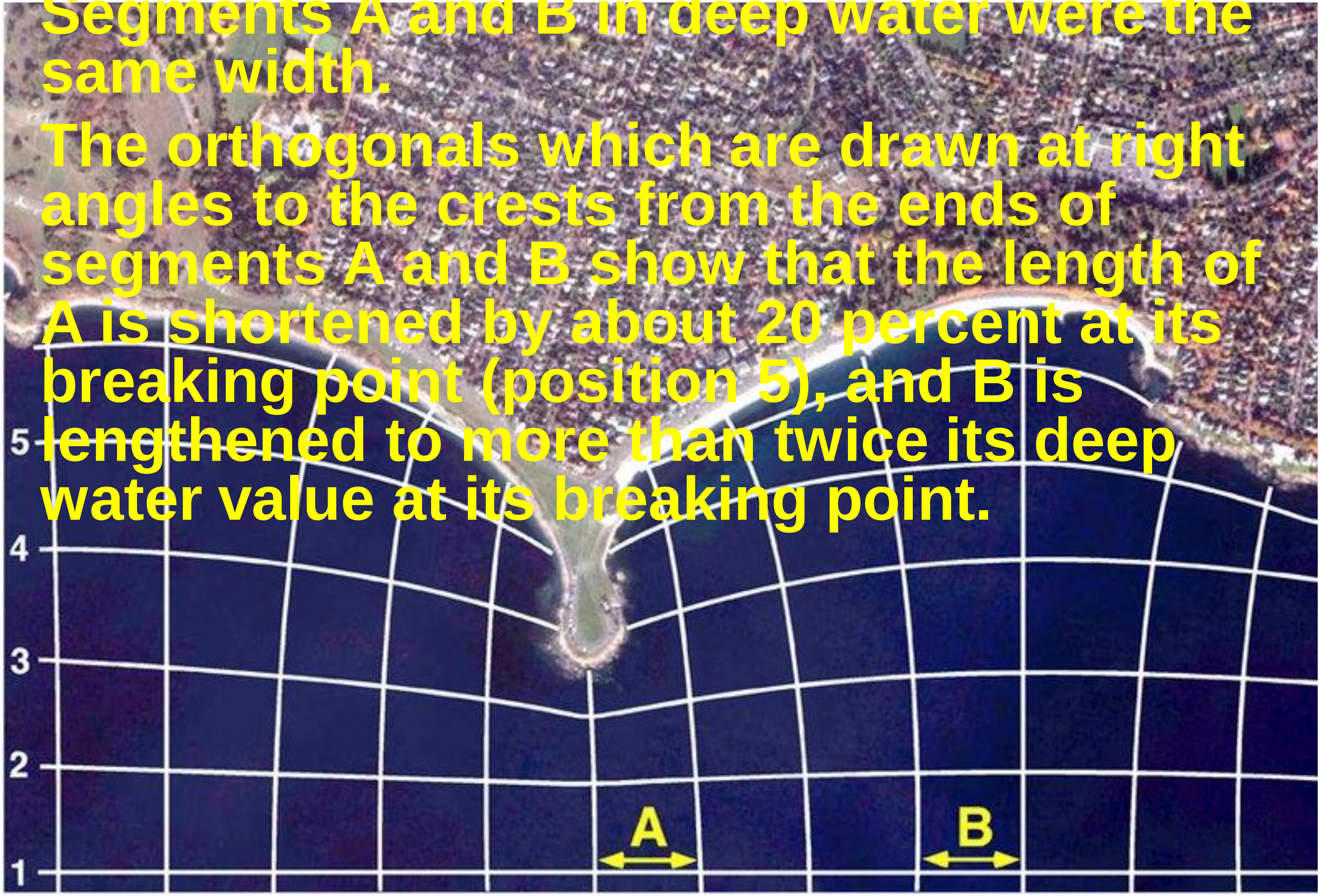


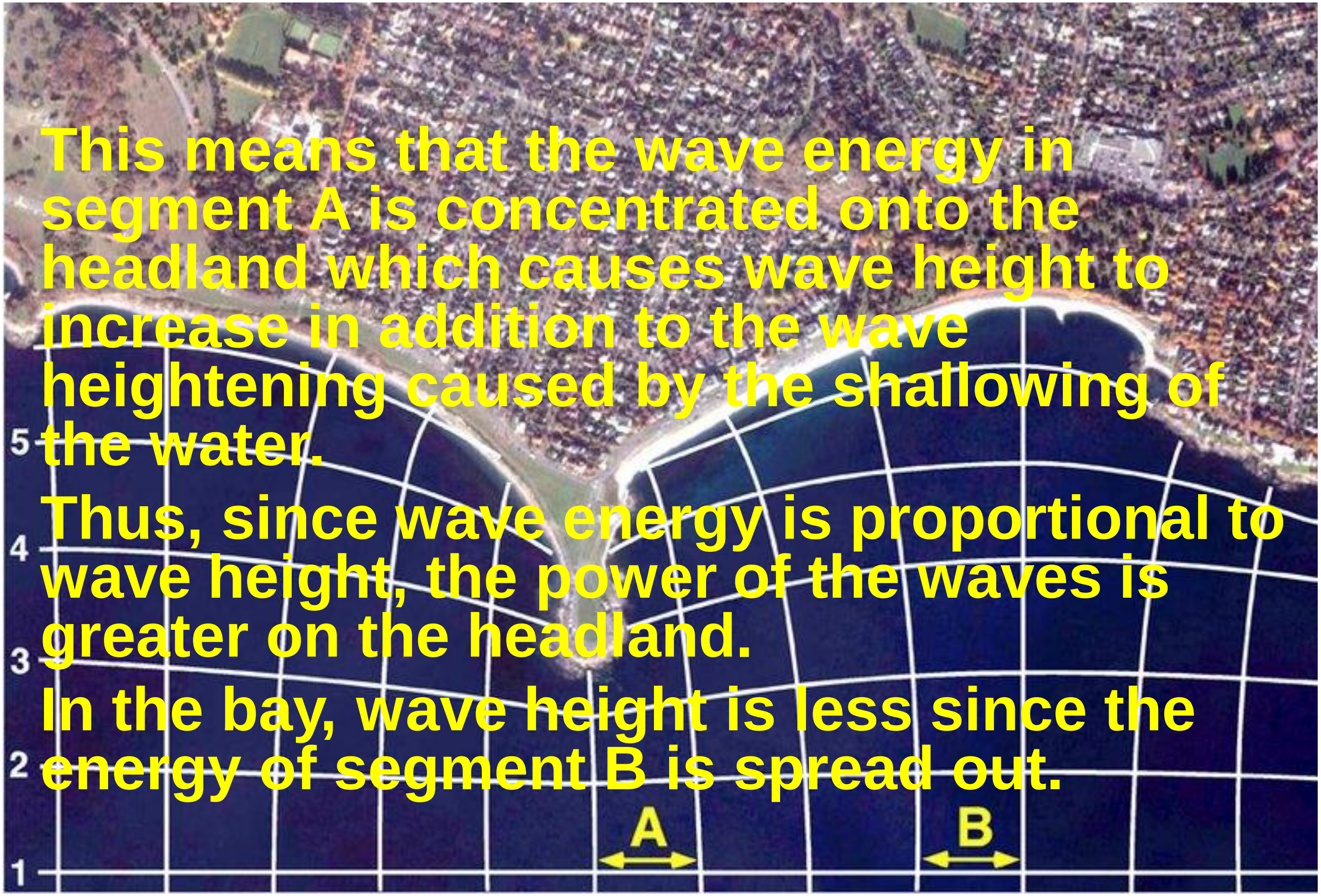
- By the time the wave reaches position 5, A is about to break on the headland while B is advancing more slowly into the bay.

- The end result is that the crests try to conform to the outline of the shore and to break parallel to it.

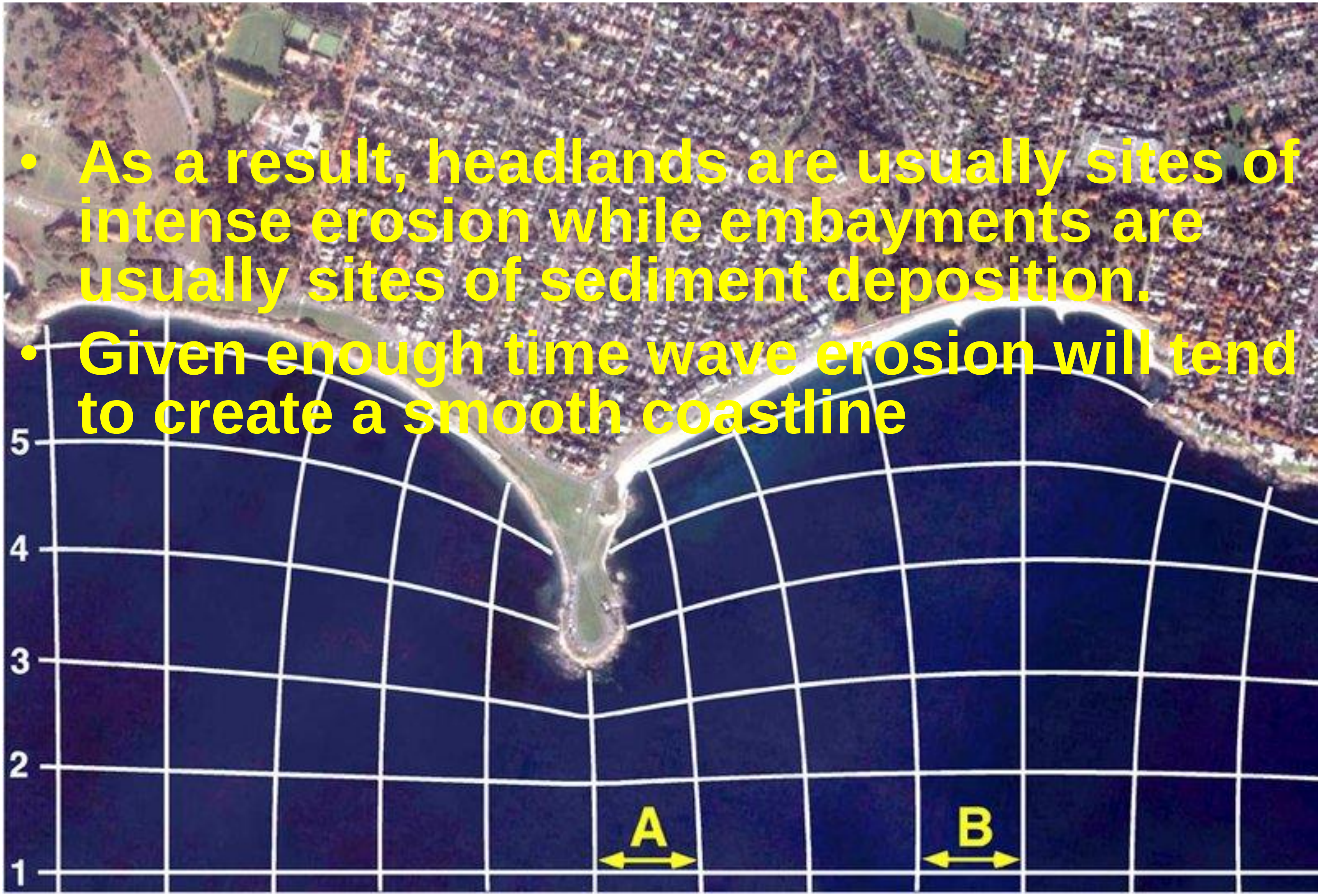


- Segments A and B in deep water were the same width.
- The orthogonals which are drawn at right angles to the crests from the ends of segments A and B show that the length of A is shortened by about 20 percent at its breaking point (position 5), and B is lengthened to more than twice its deep water value at its breaking point.



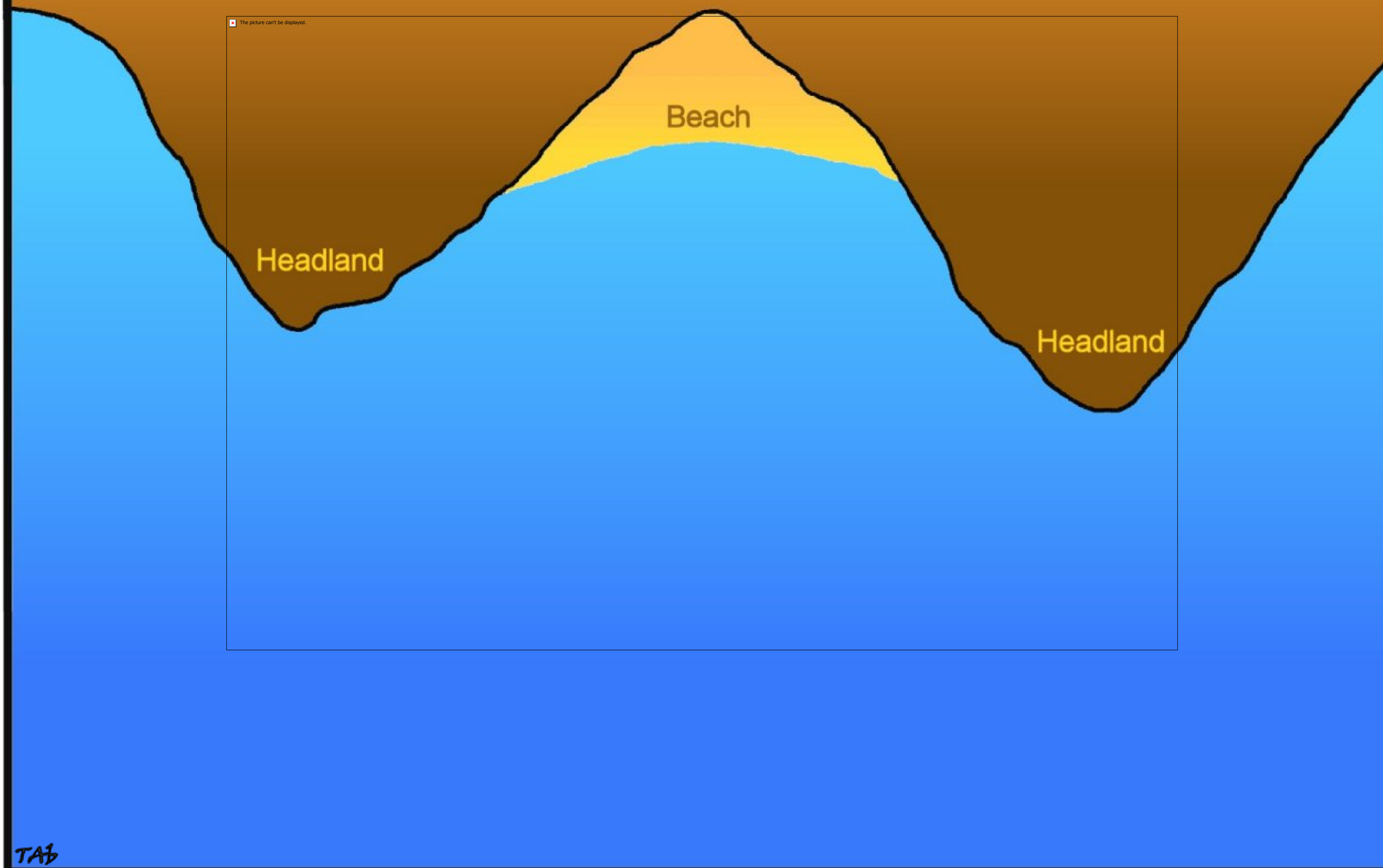


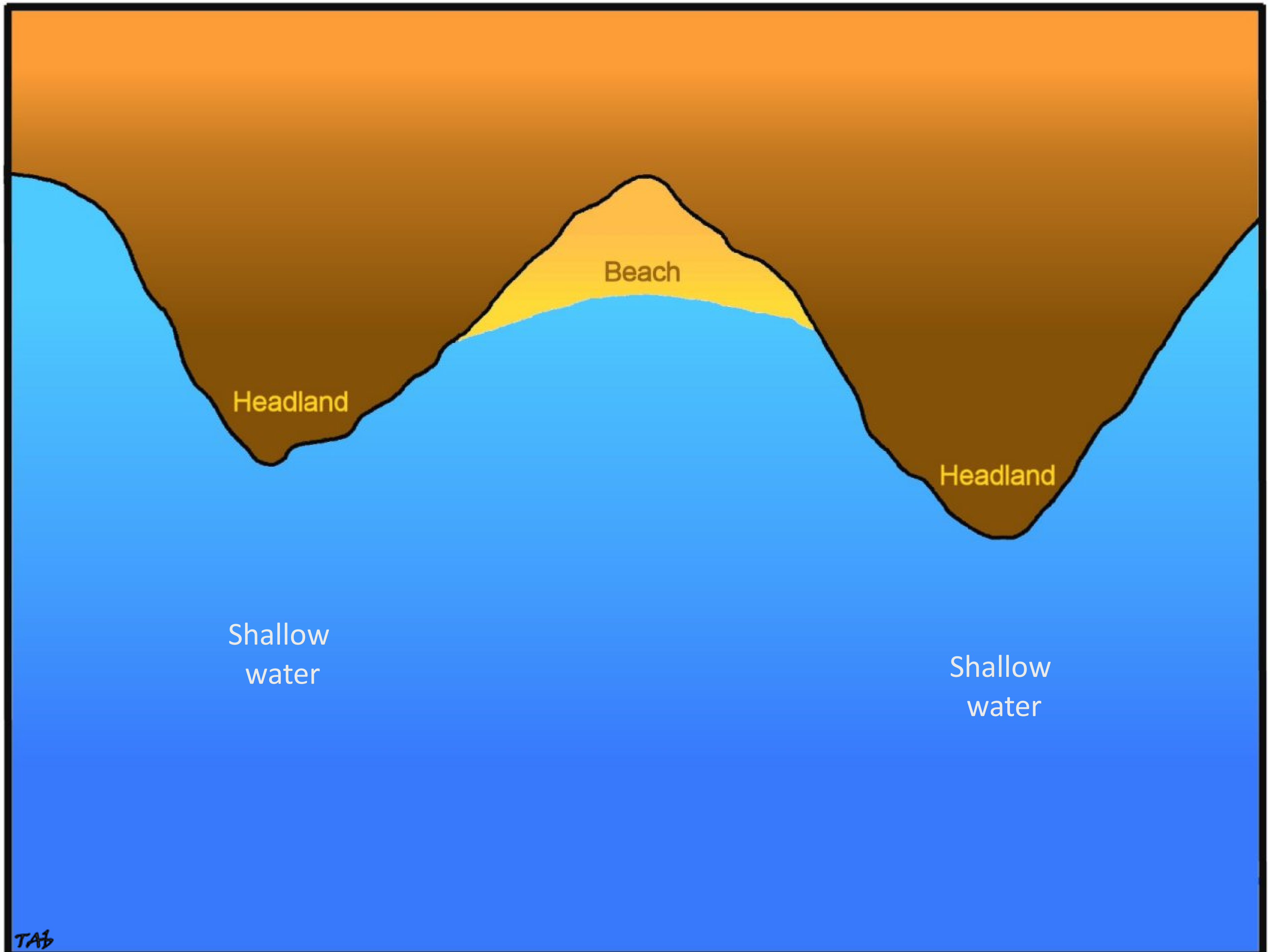
- This means that the wave energy in segment A is concentrated onto the headland which causes wave height to increase in addition to the wave heightening caused by the shallowing of the water.
- Thus, since wave energy is proportional to wave height, the power of the waves is greater on the headland.
- In the bay, wave height is less since the energy of segment B is spread out.

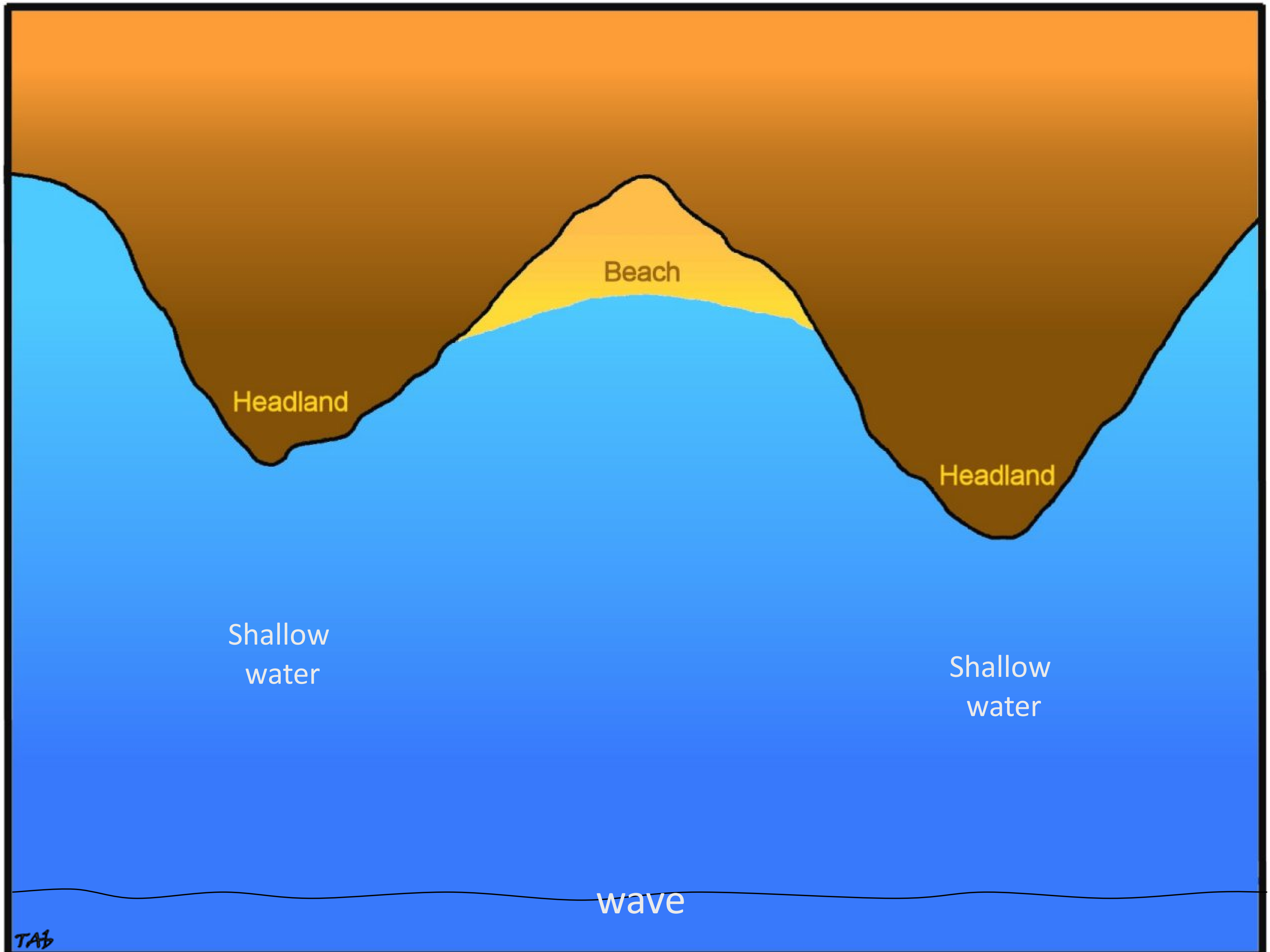


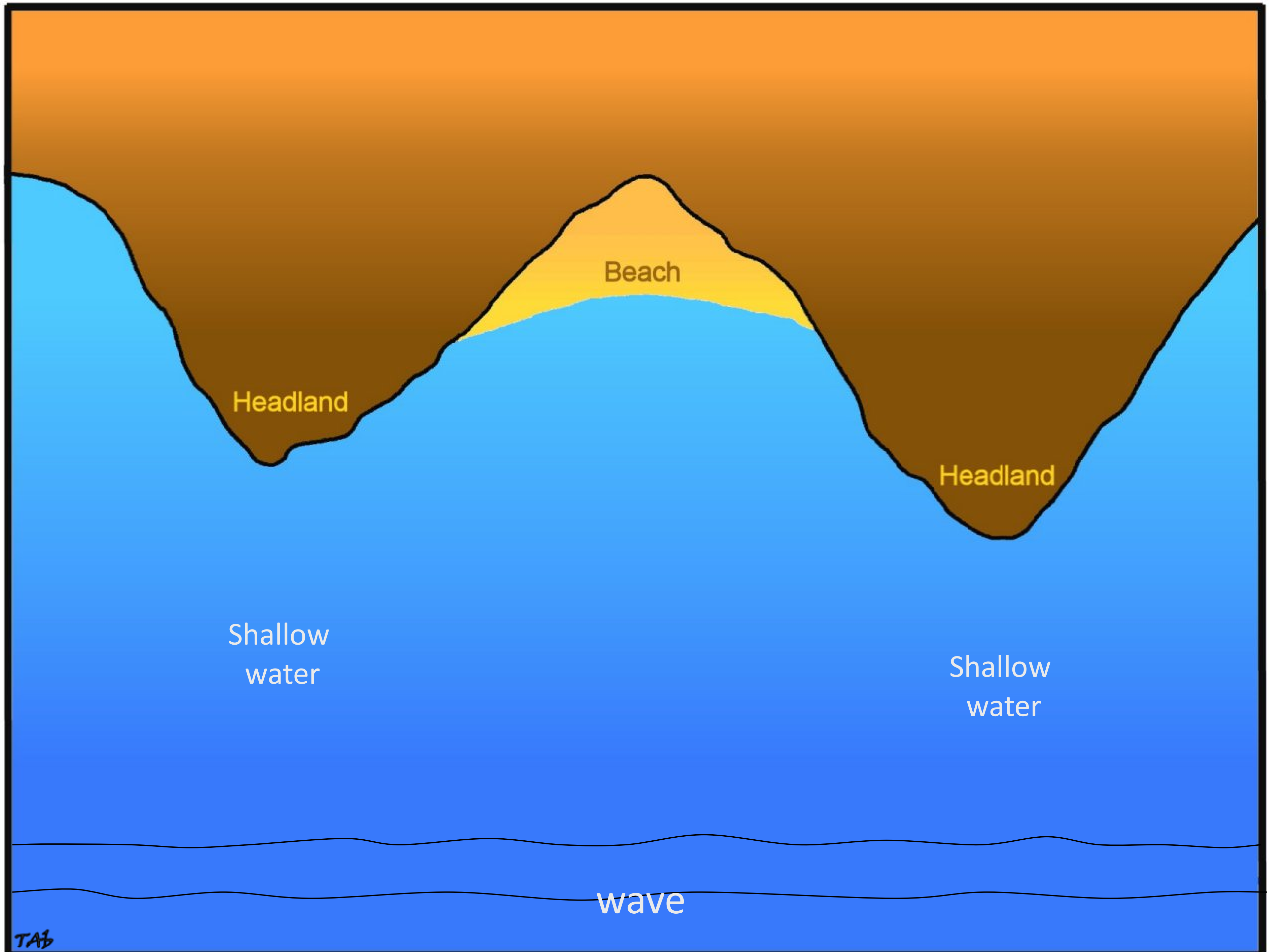
- As a result, headlands are usually sites of intense erosion while embayments are usually sites of sediment deposition.
- Given enough time wave erosion will tend to create a smooth coastline

Wave Refraction and Orthogonals




Beach
Headland
Headland
Shallow water
Shallow water
TAb


Beach
Headland
Headland
Shallow water
Shallow water
wave
TAB


Beach
Headland
Headland
Shallow water
Shallow water
wave
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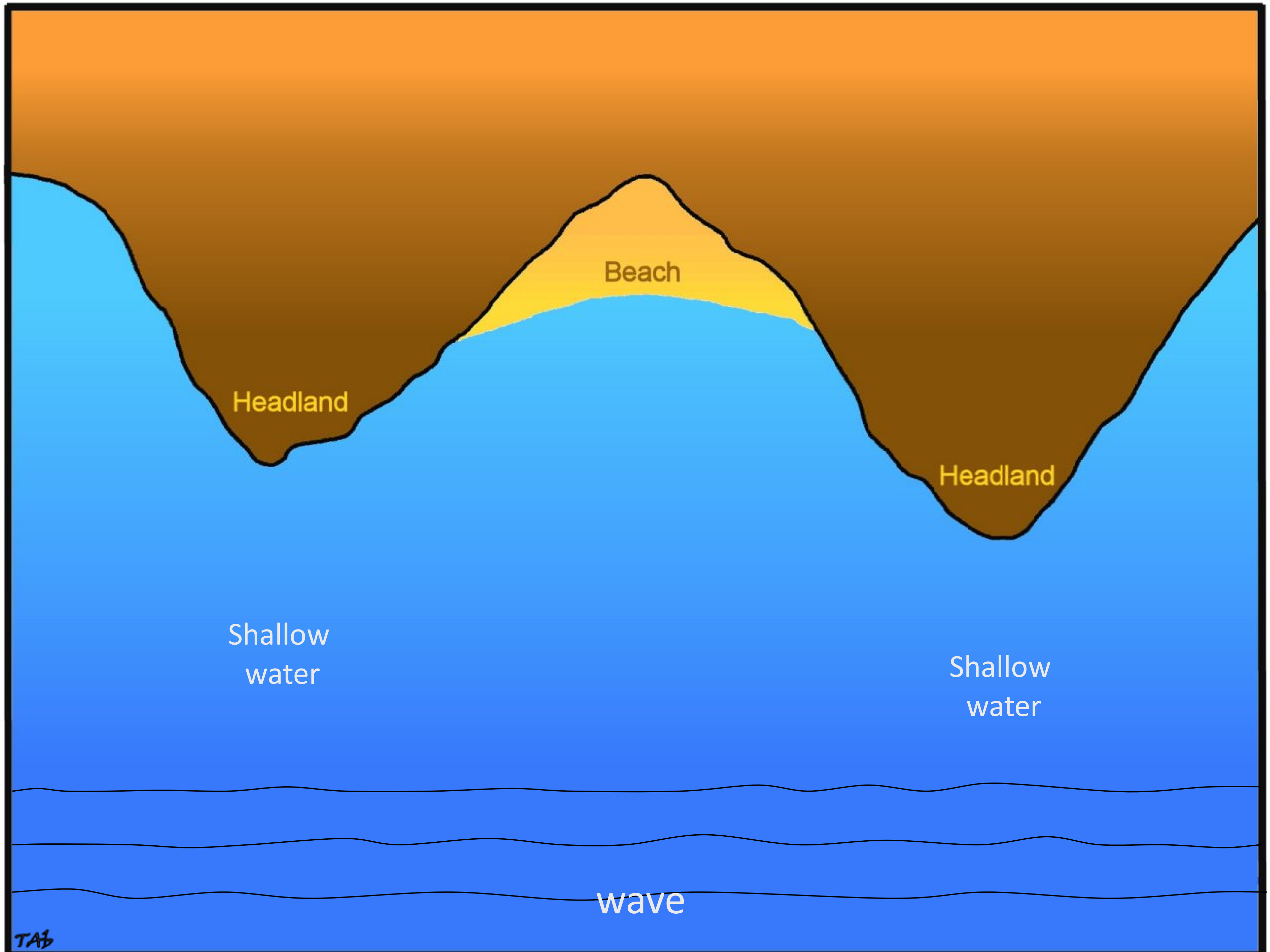
Beach
Headland
Headland
Shallow water
Shallow water
wave
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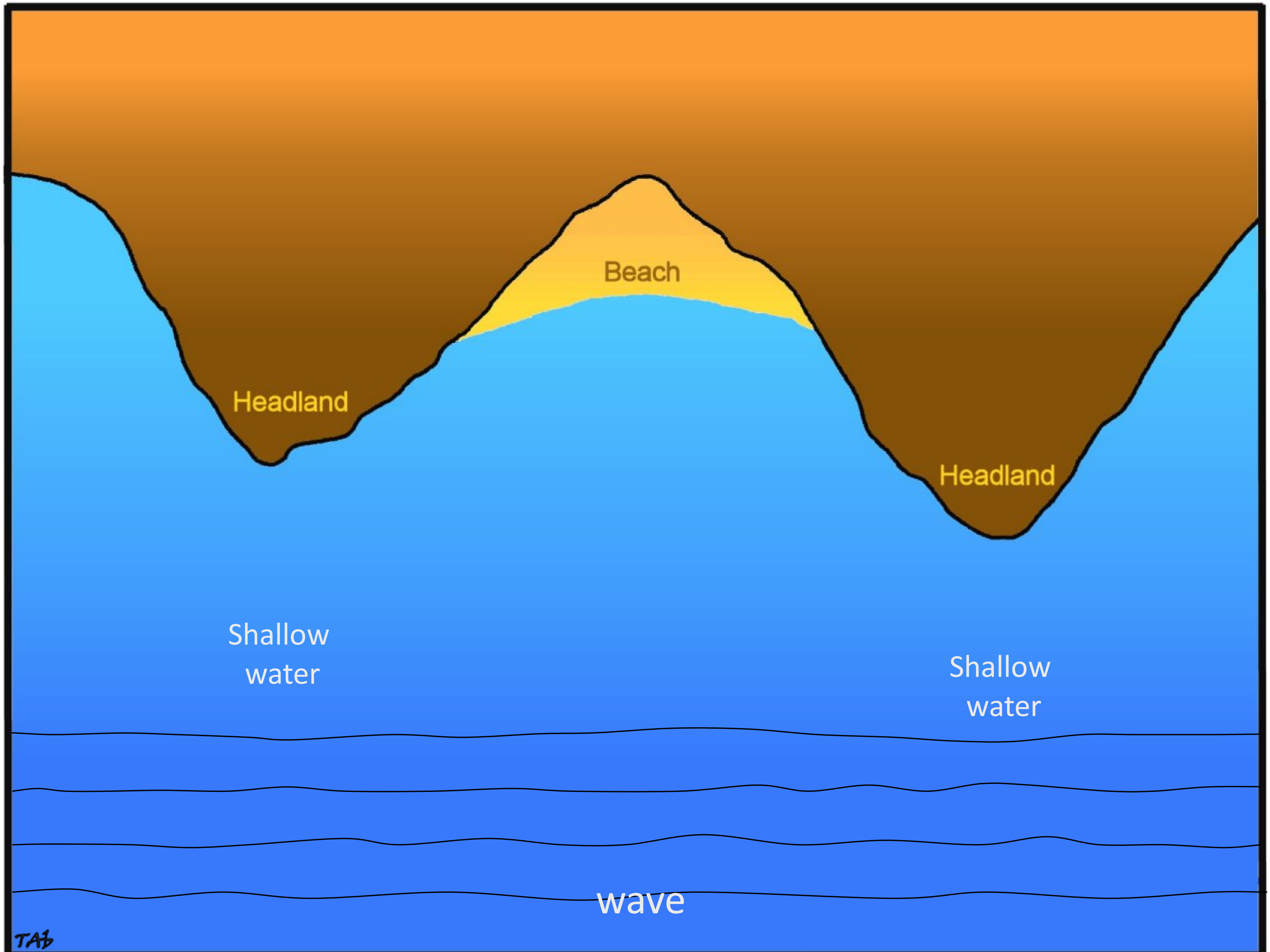
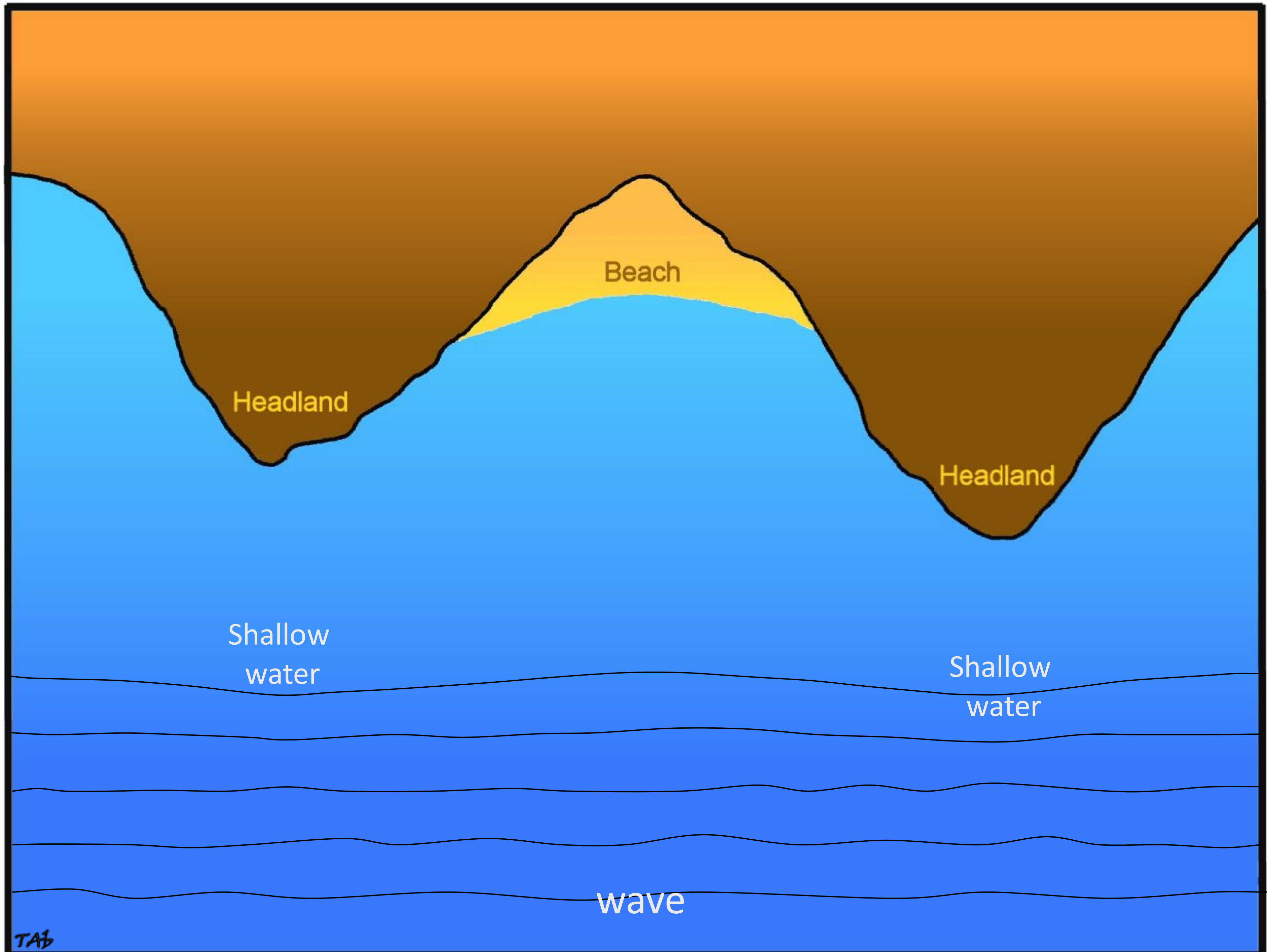
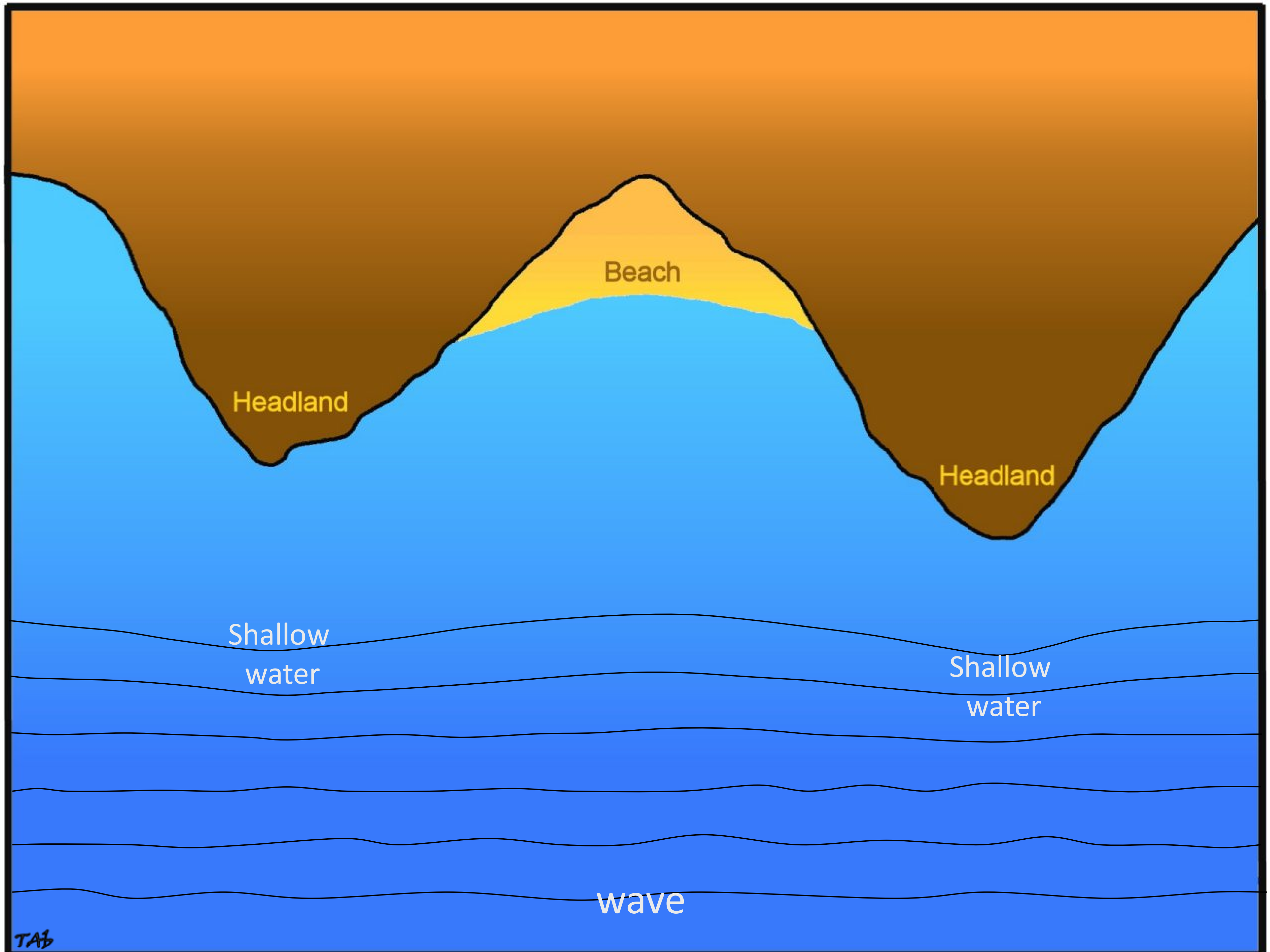

Beach
Headland
Headland
Shallow water
Shallow water
wave
TAb



Beach
Headland
Headland
Shallow water
Shallow water
wave
TAb


Beach
Headland
Headland
Shallow water
Shallow water
wave
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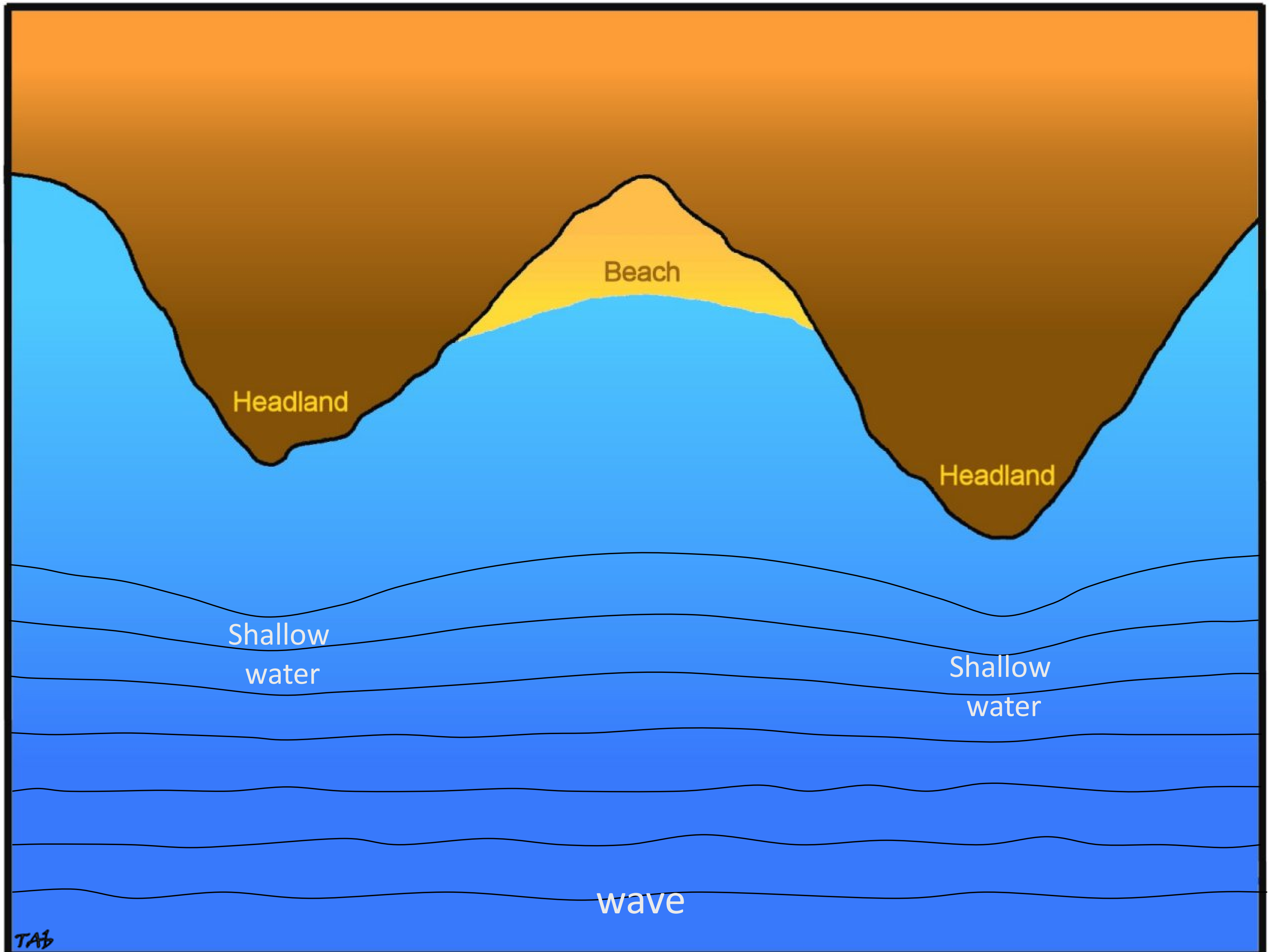
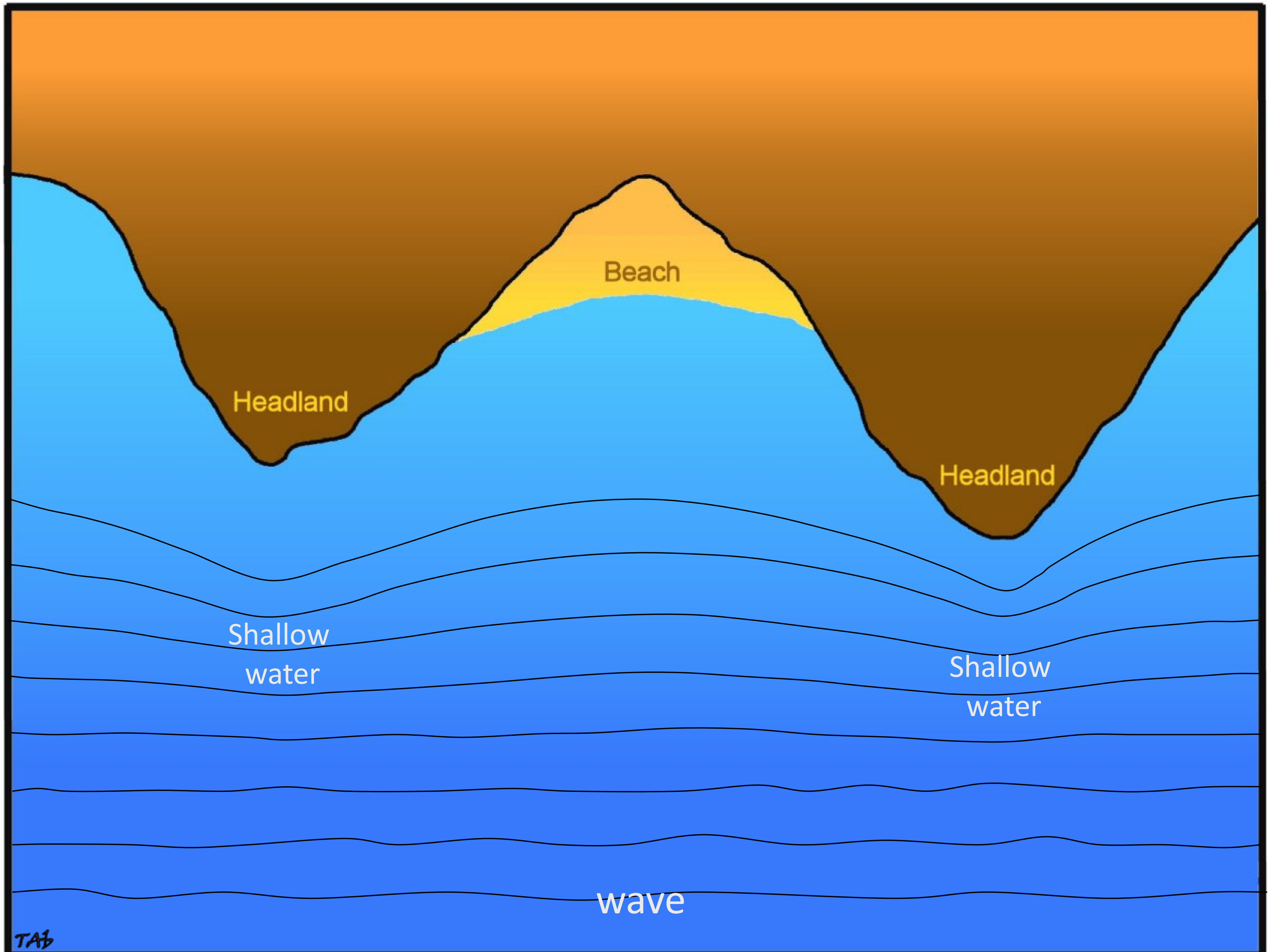
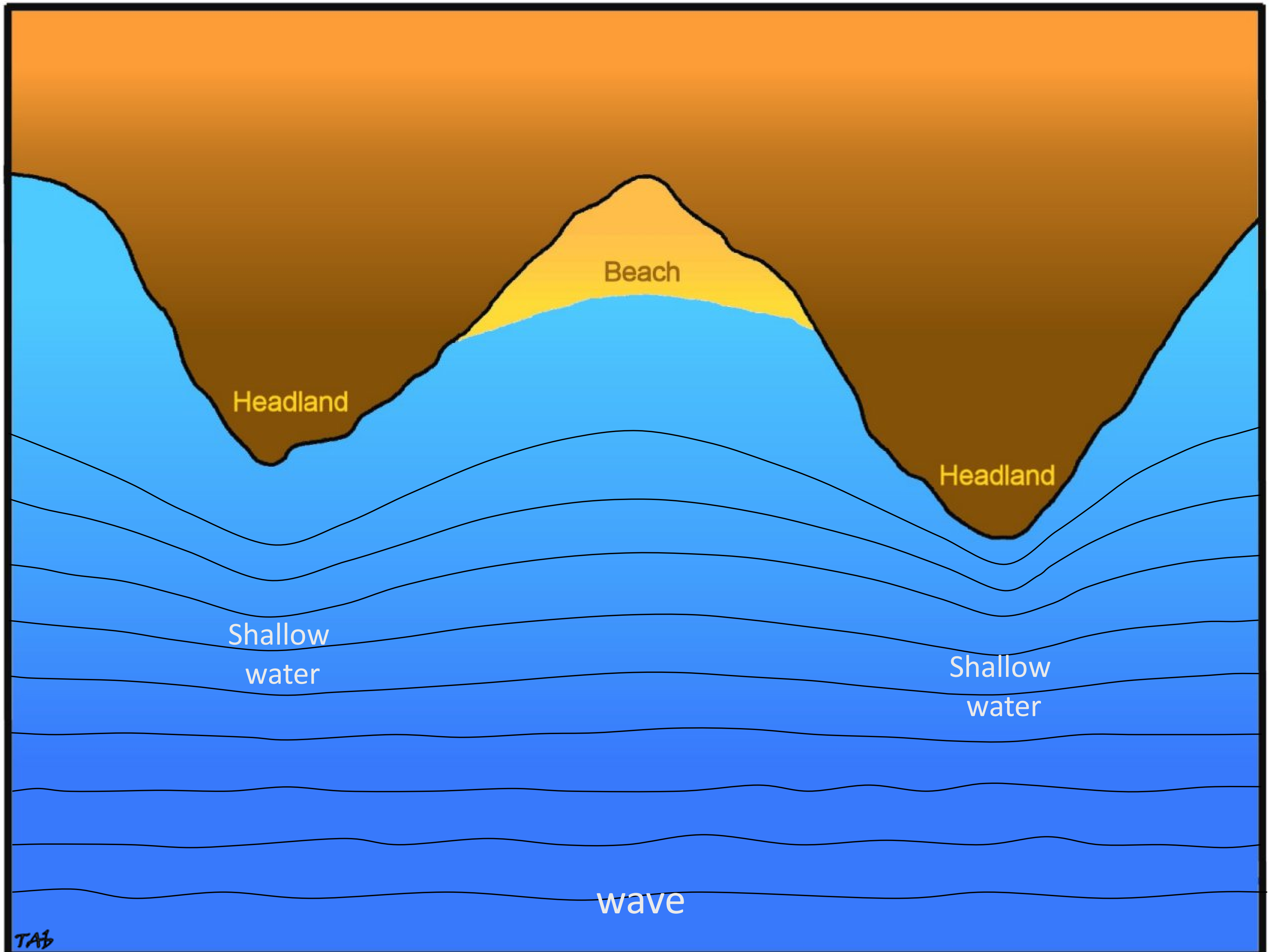
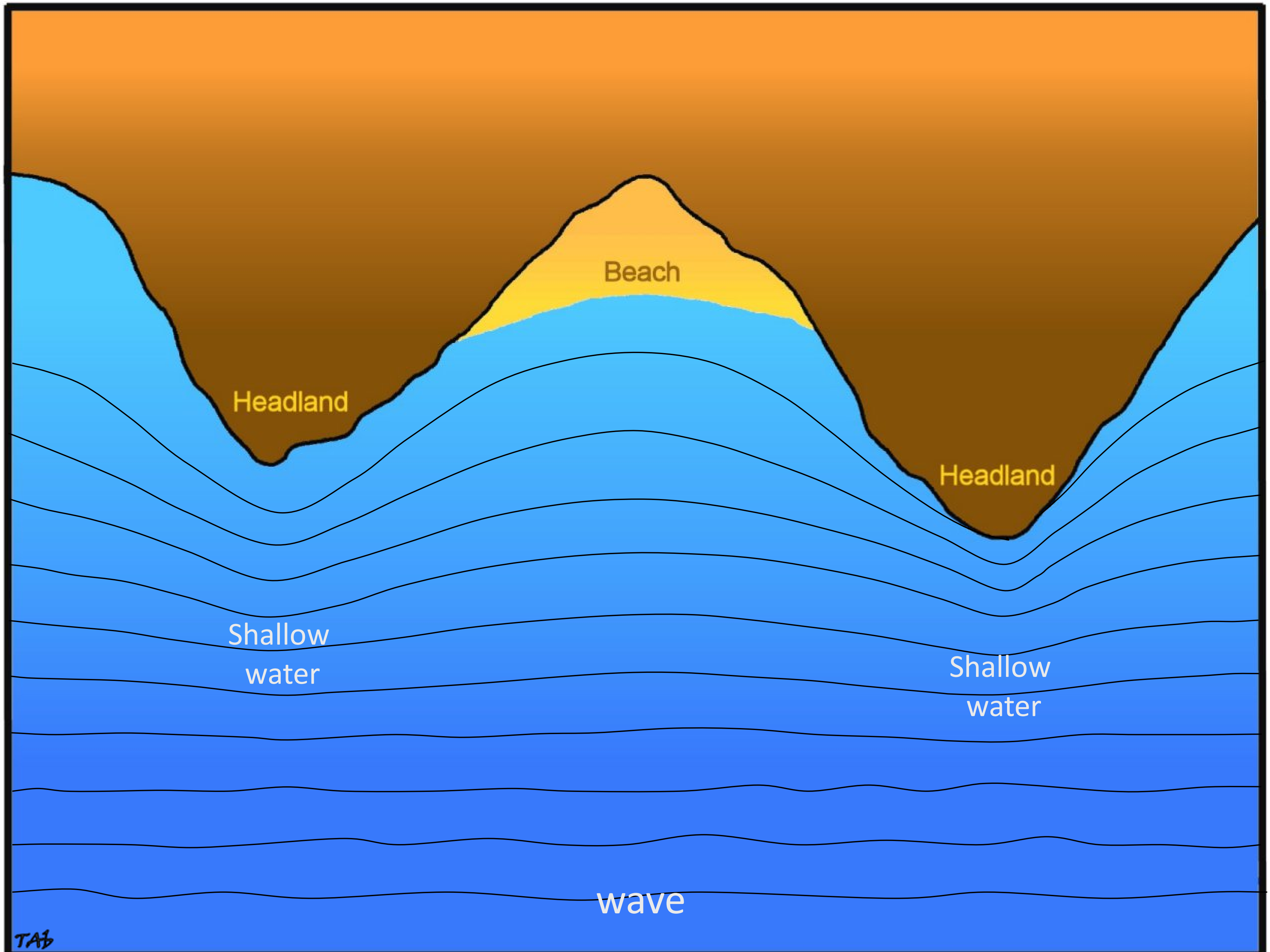
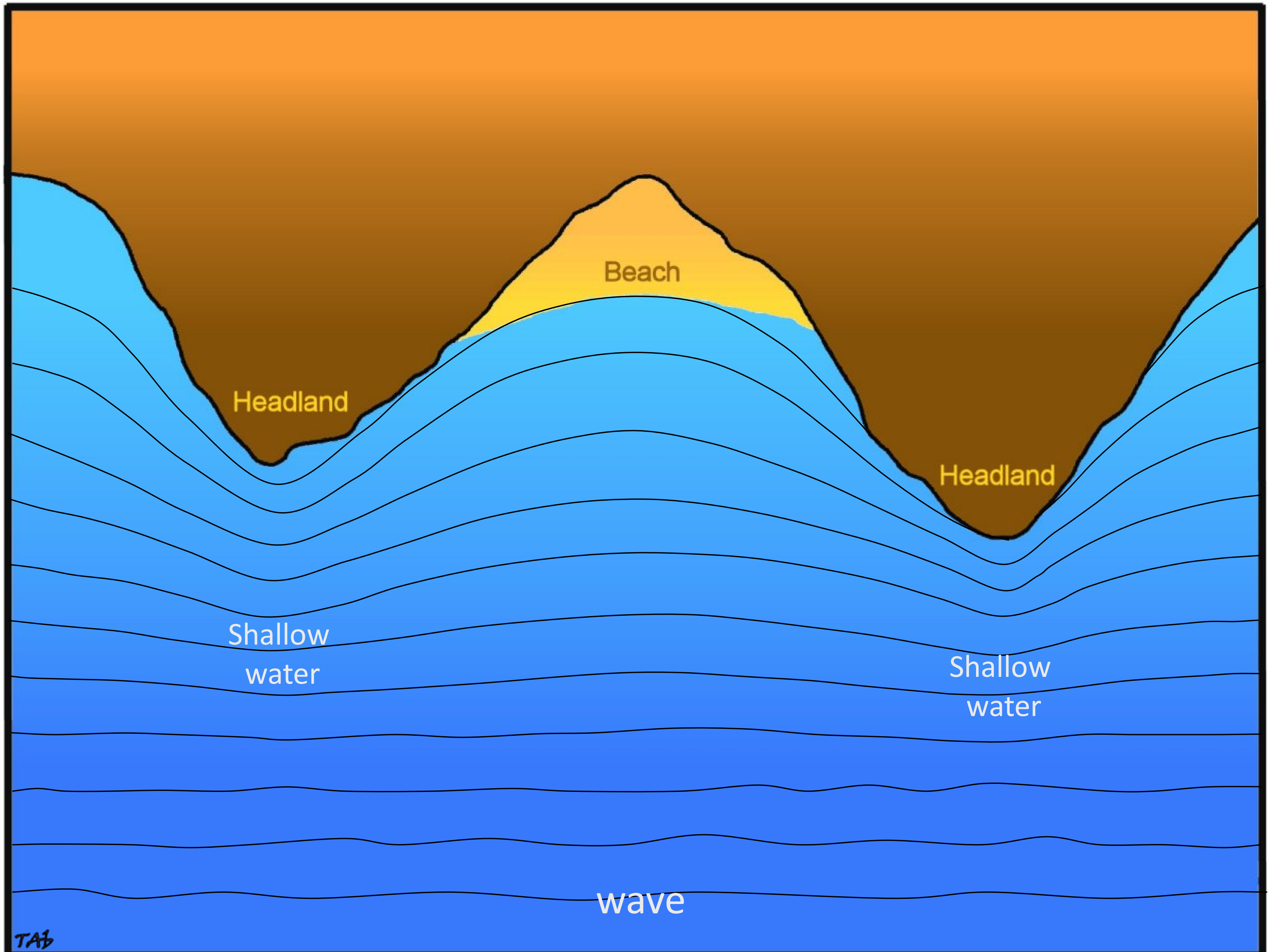
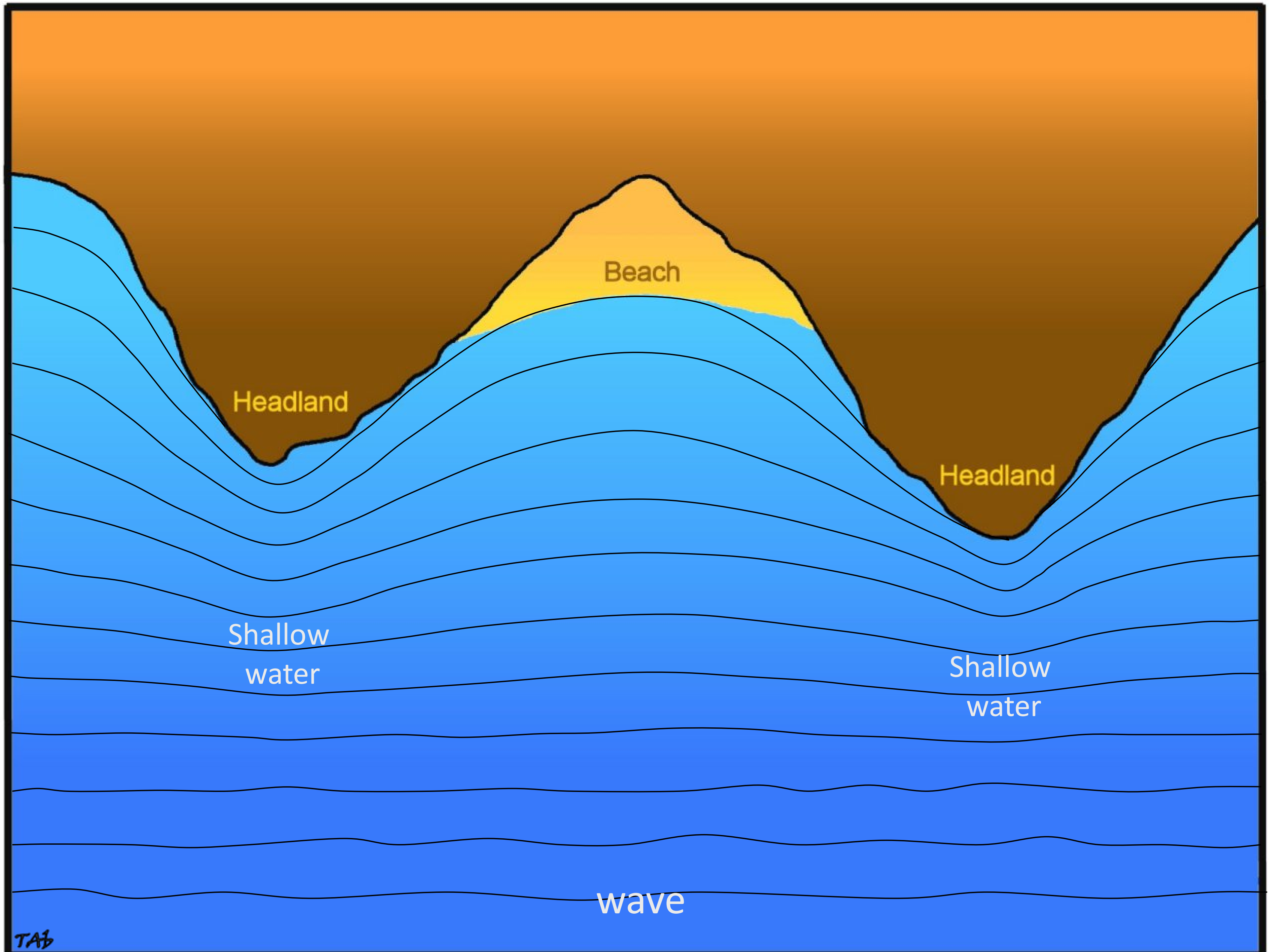

Beach
Headland
Headland
Shallow
water
Shallow
water
wave
TAB

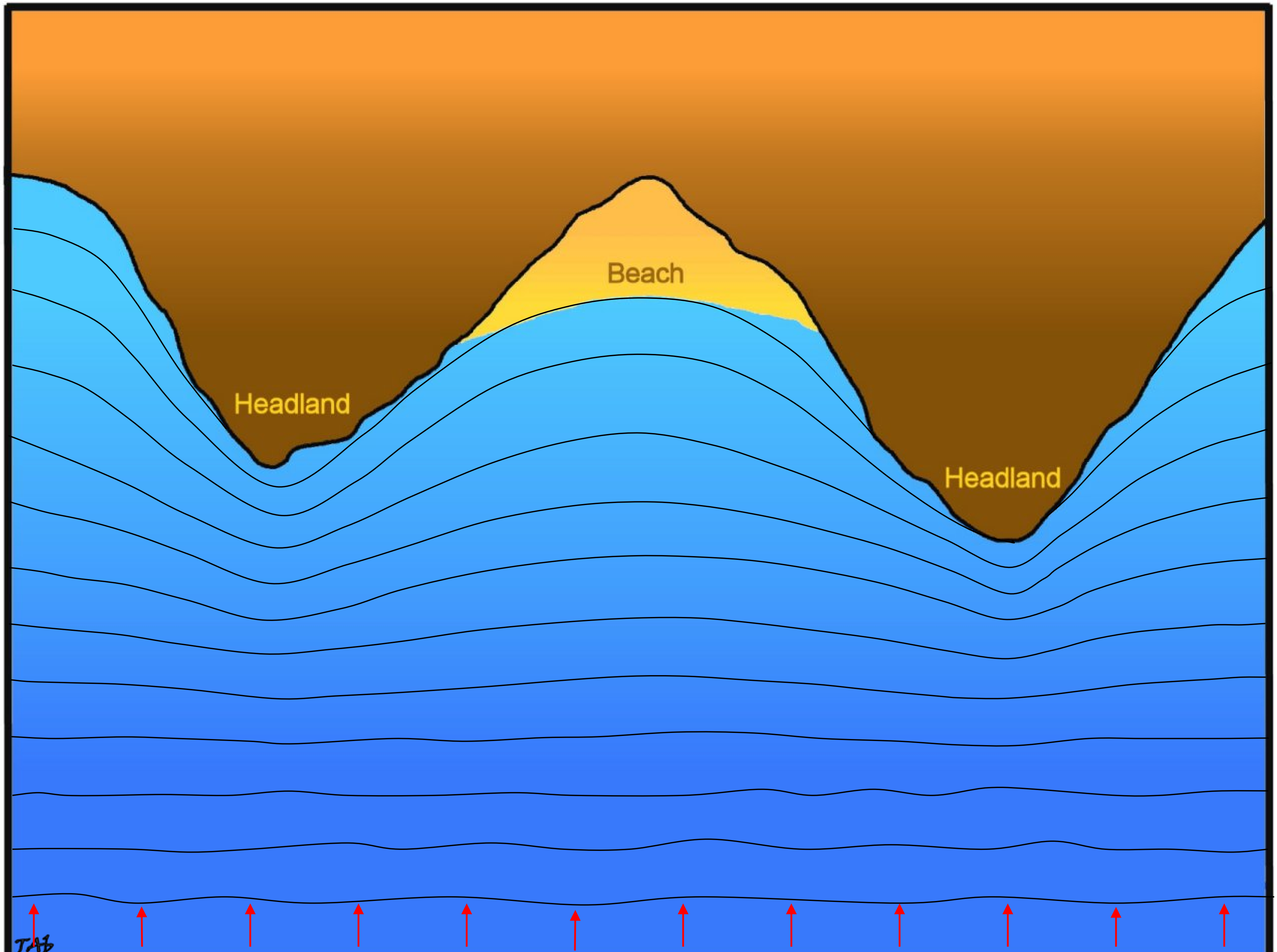

Beach
Headland
Headland
Shallow water
Shallow water
wave
TAb

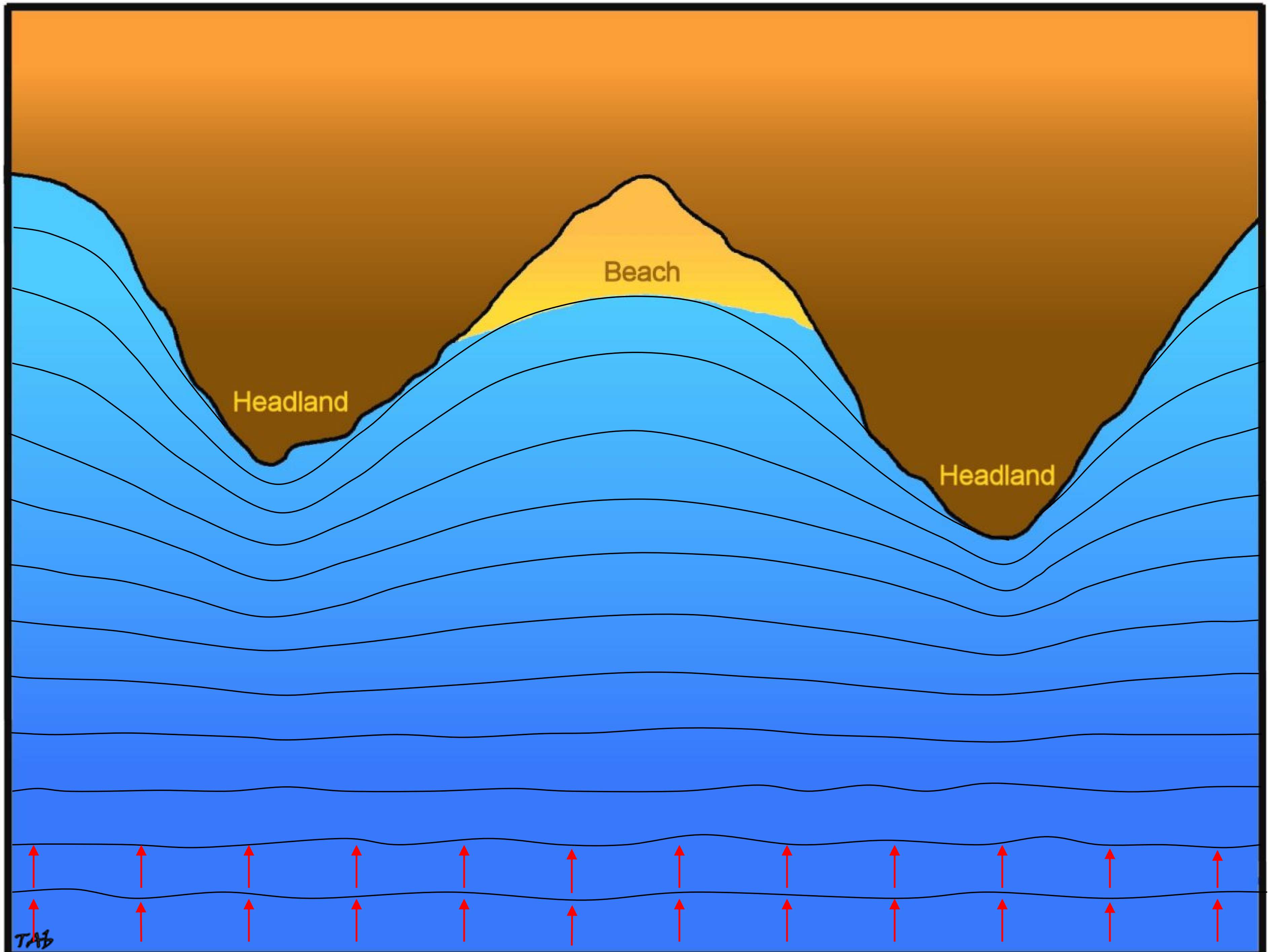

Beach
Headland
Headland
Shallow
water
Shallow
water
wave
TAB

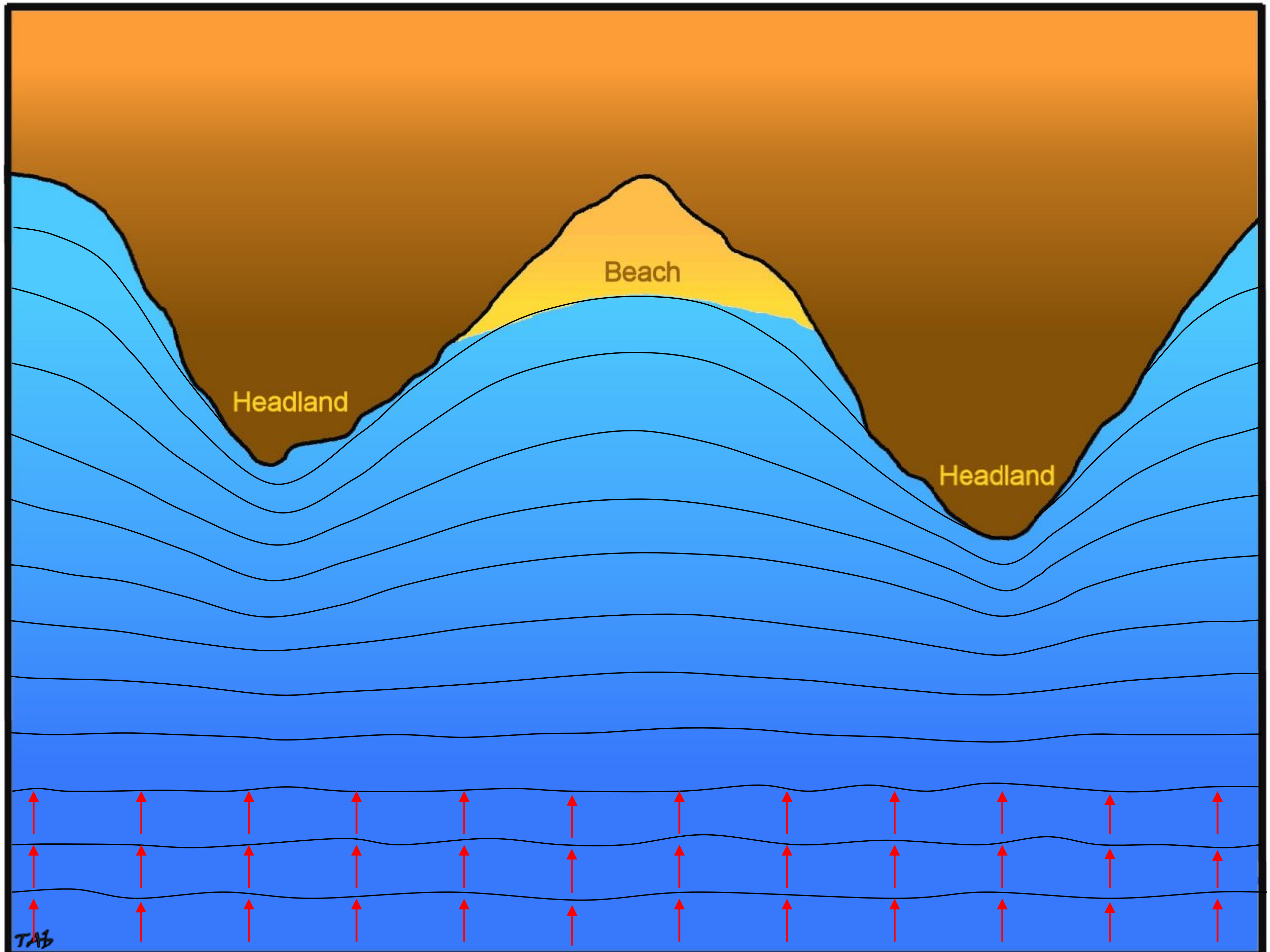

Beach
Headland
Headland
Shallow water
Shallow water
wave
TAB

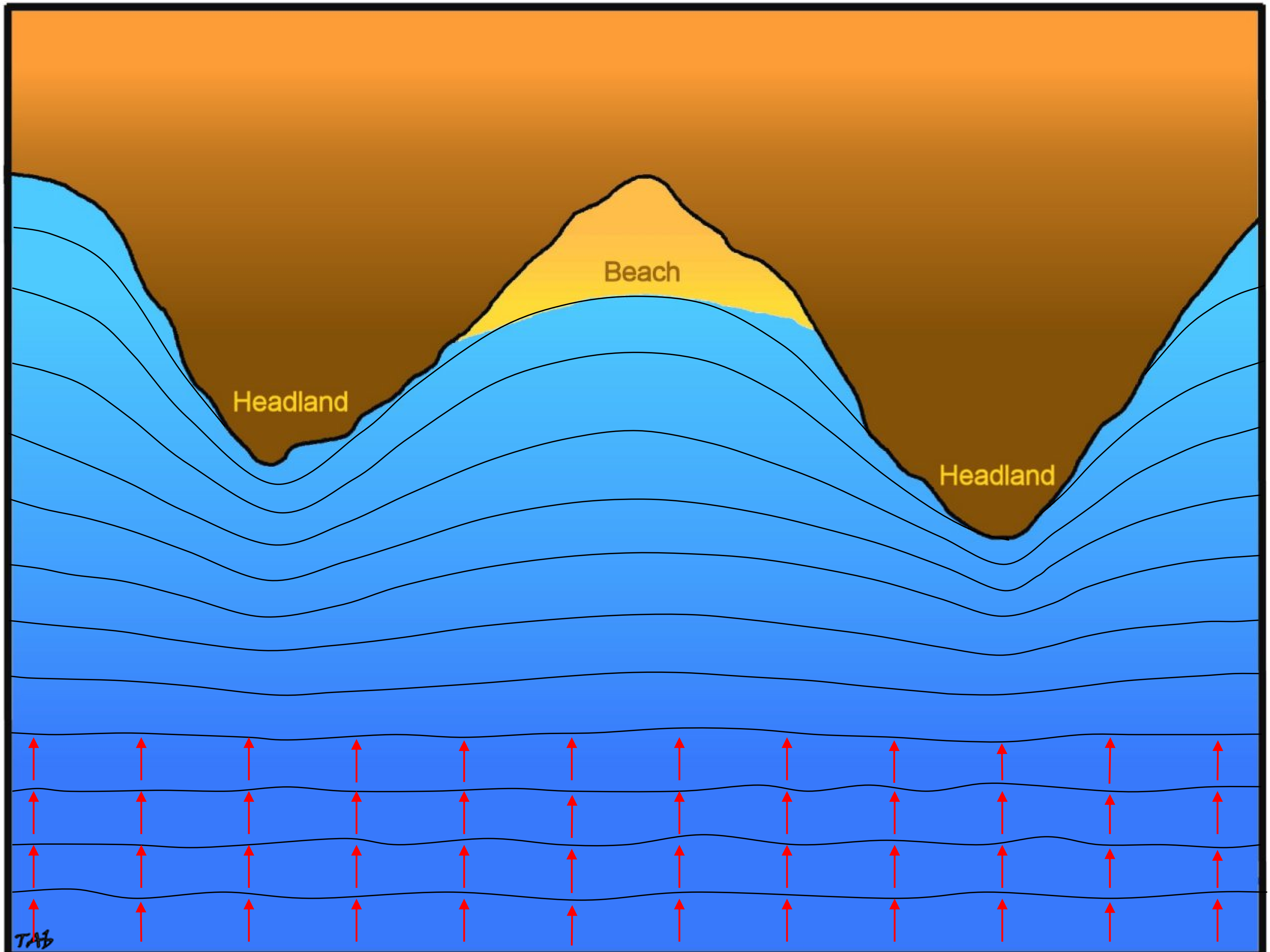

Beach
Headland
Headland
Shallow water
Shallow water
wave
TAb

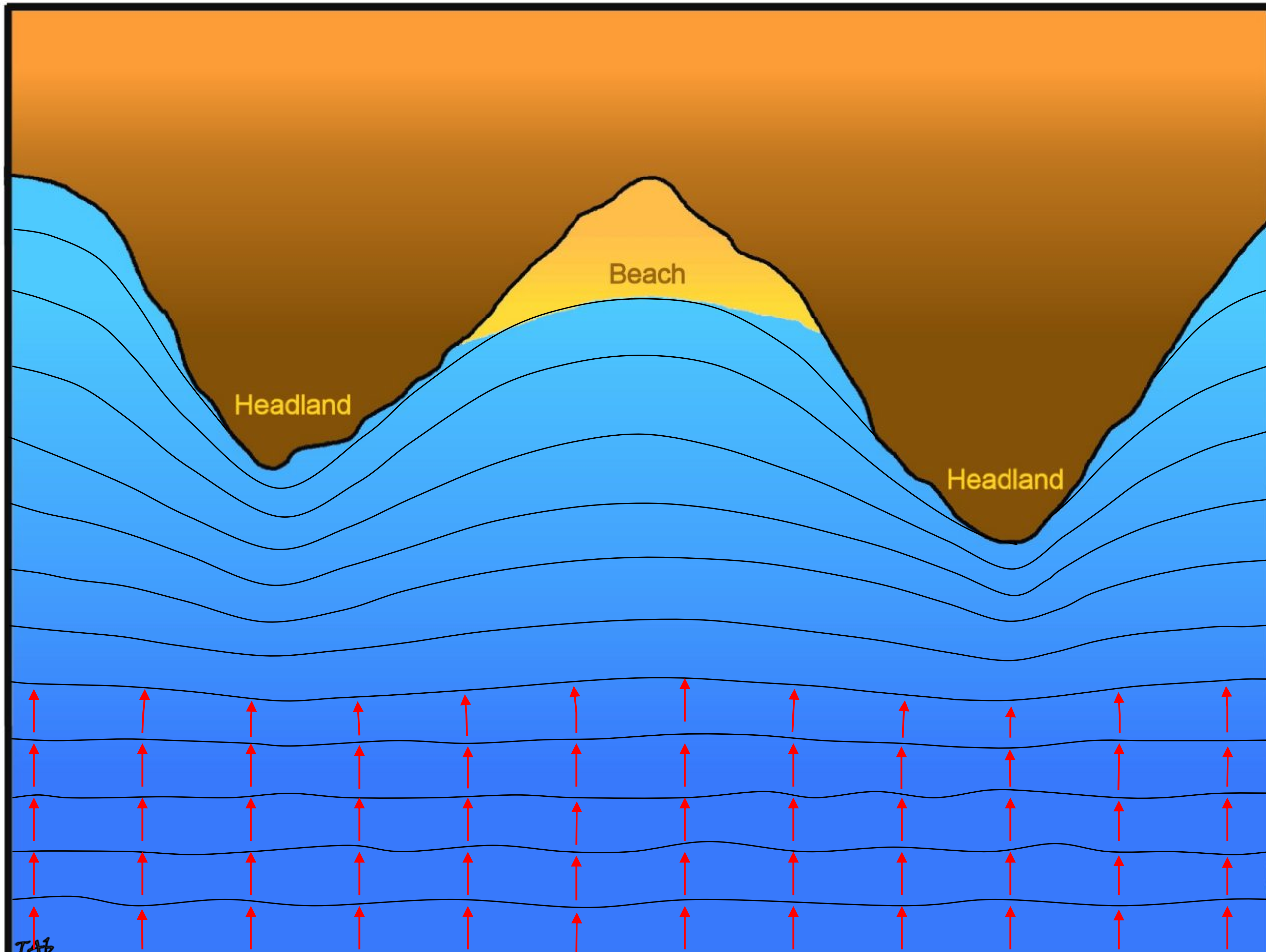

Beach
Headland
Headland
Shallow water
Shallow water
wave
TAb

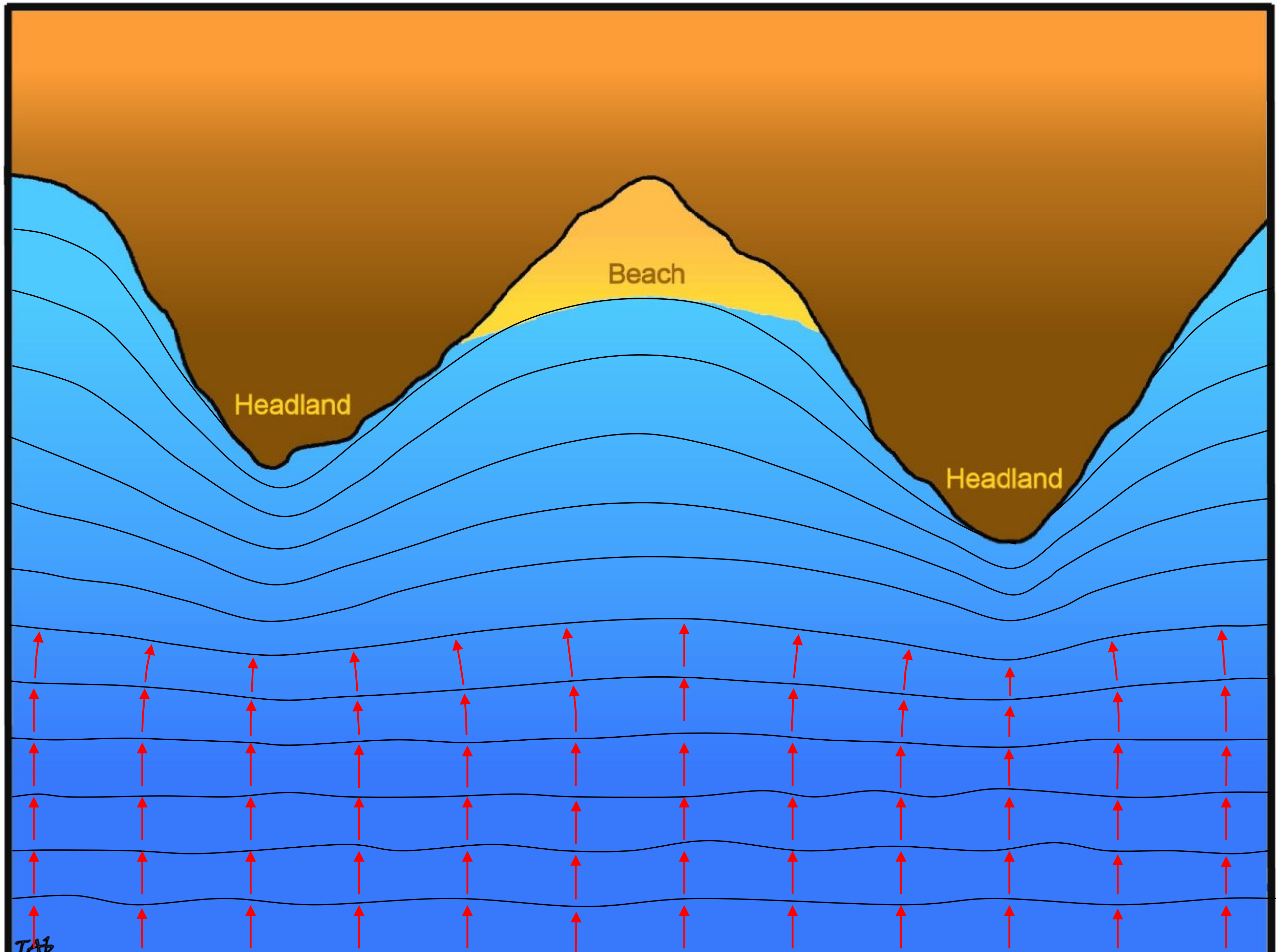

Beach
Headland
Headland
TAB

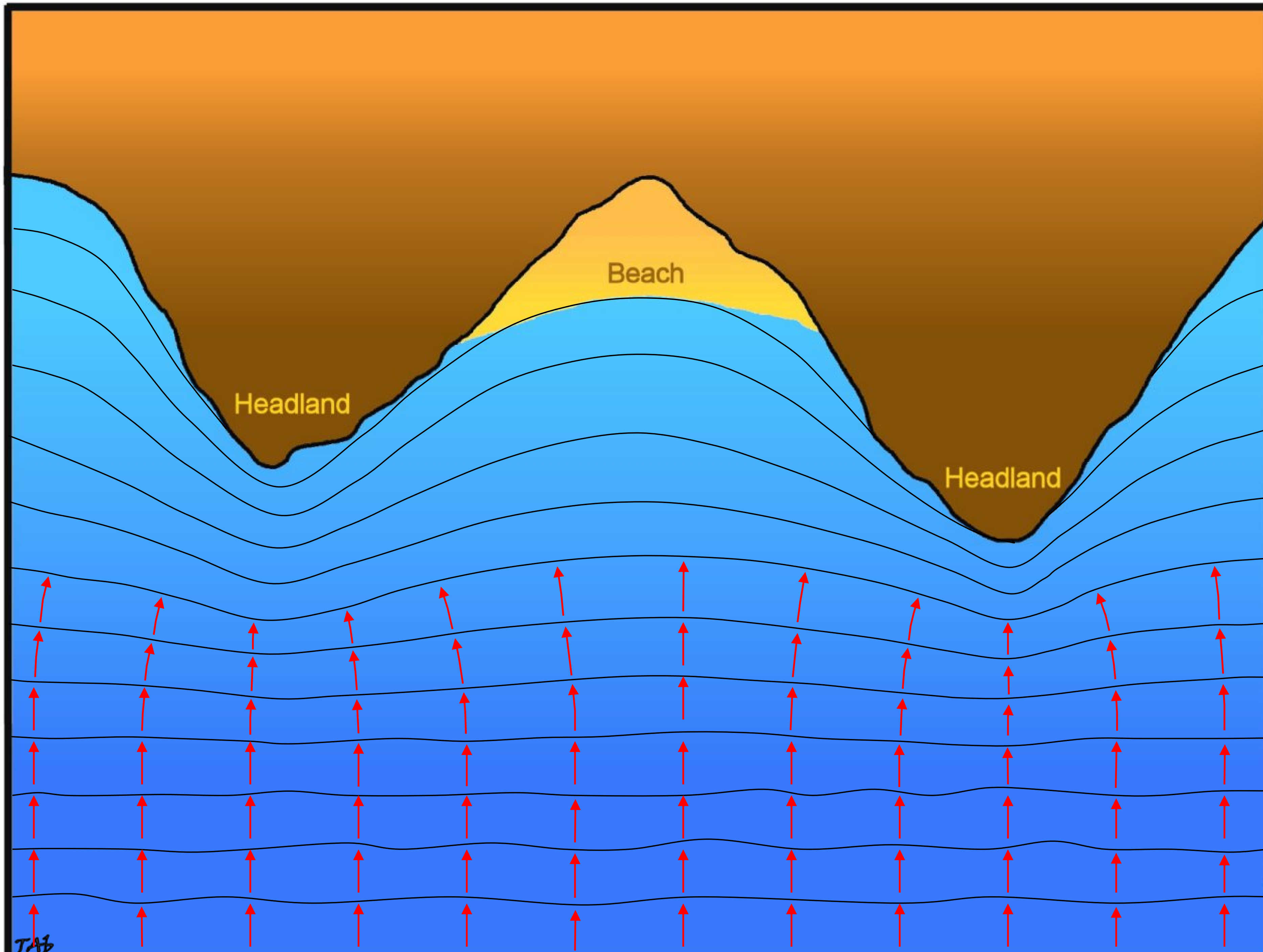

Beach
Headland
Headland

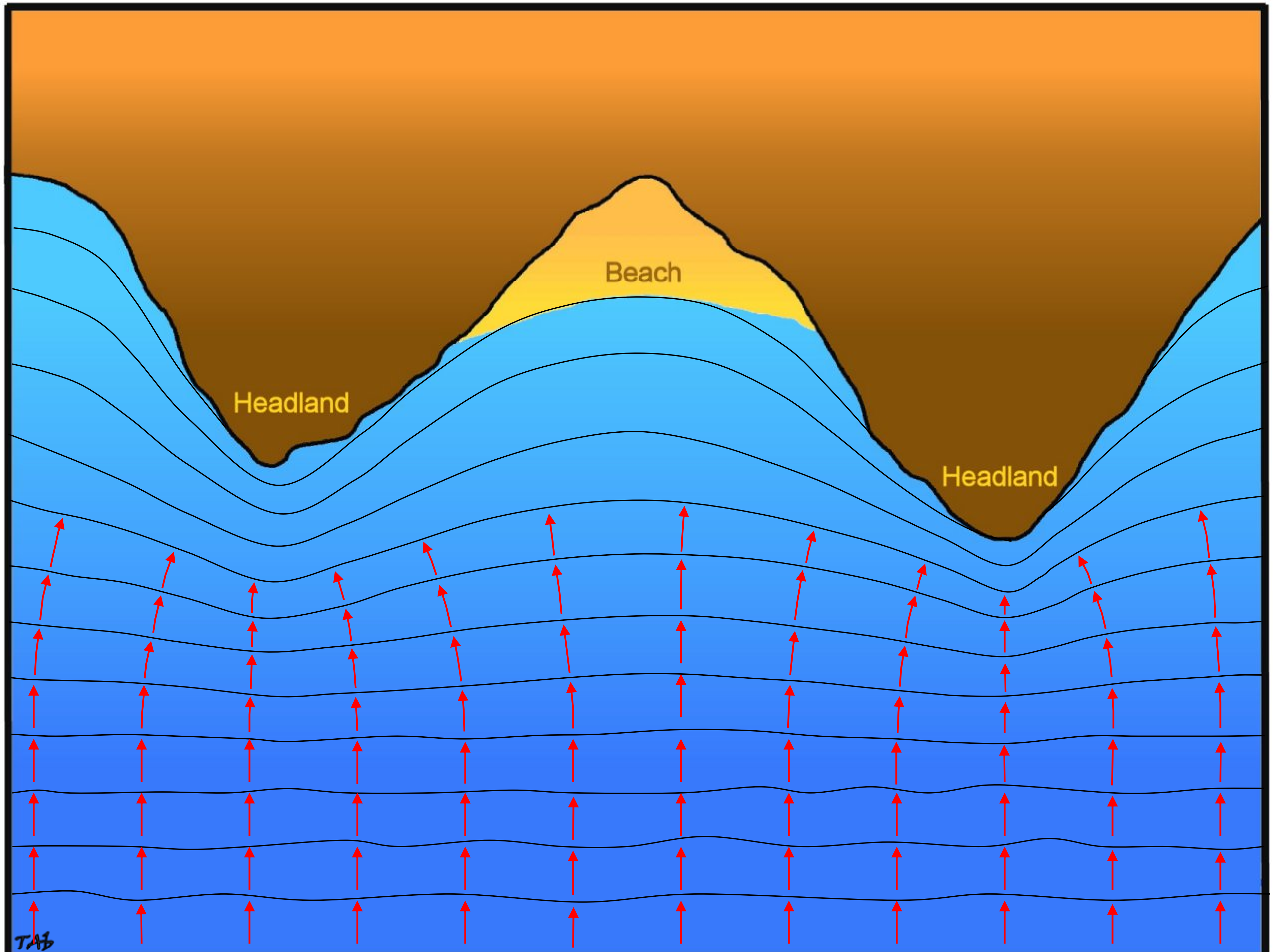

Beach
Headland
Headland
TAB

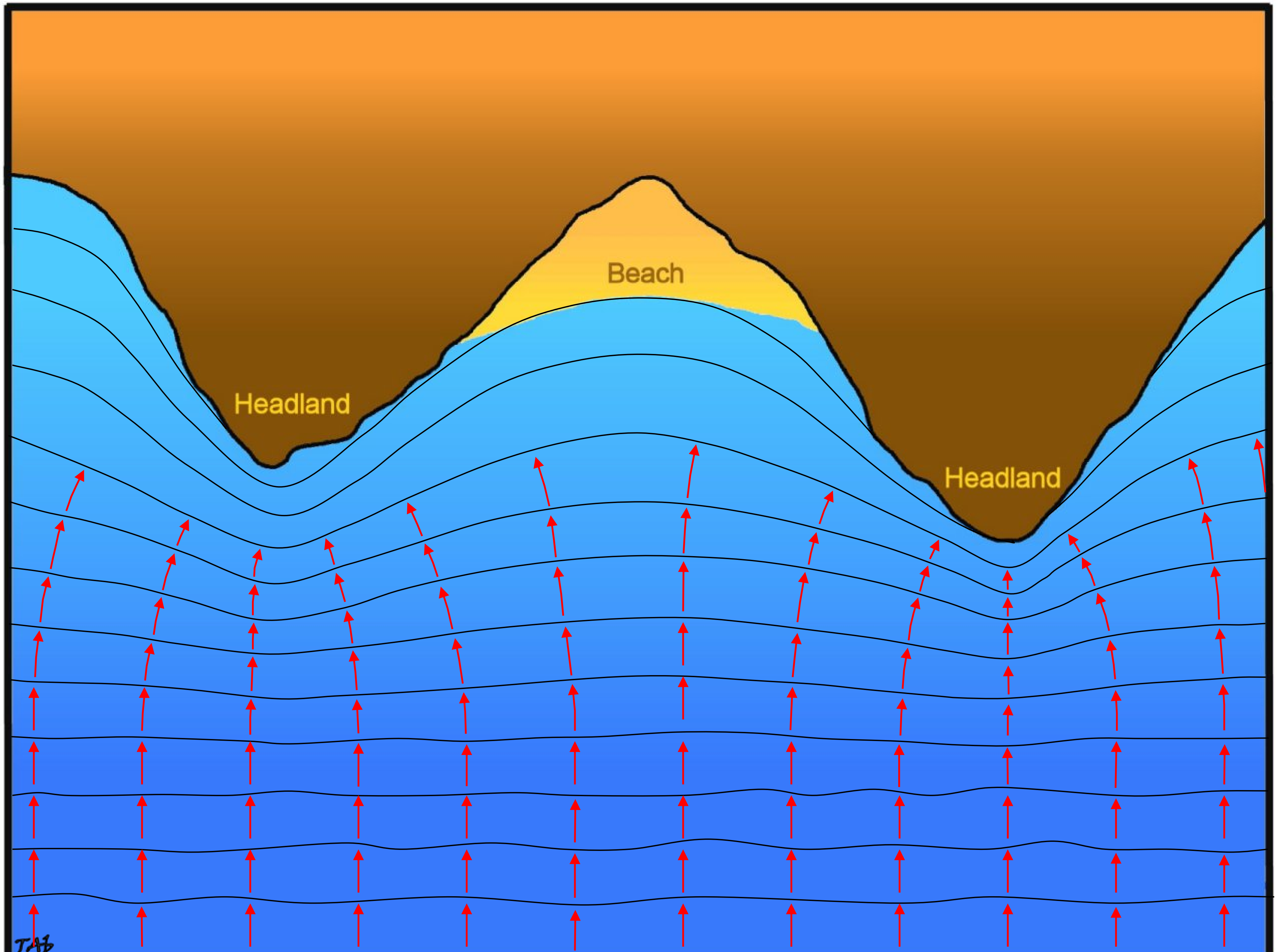

Beach
Headland
Headland
TAB


Beach
Headland
Headland

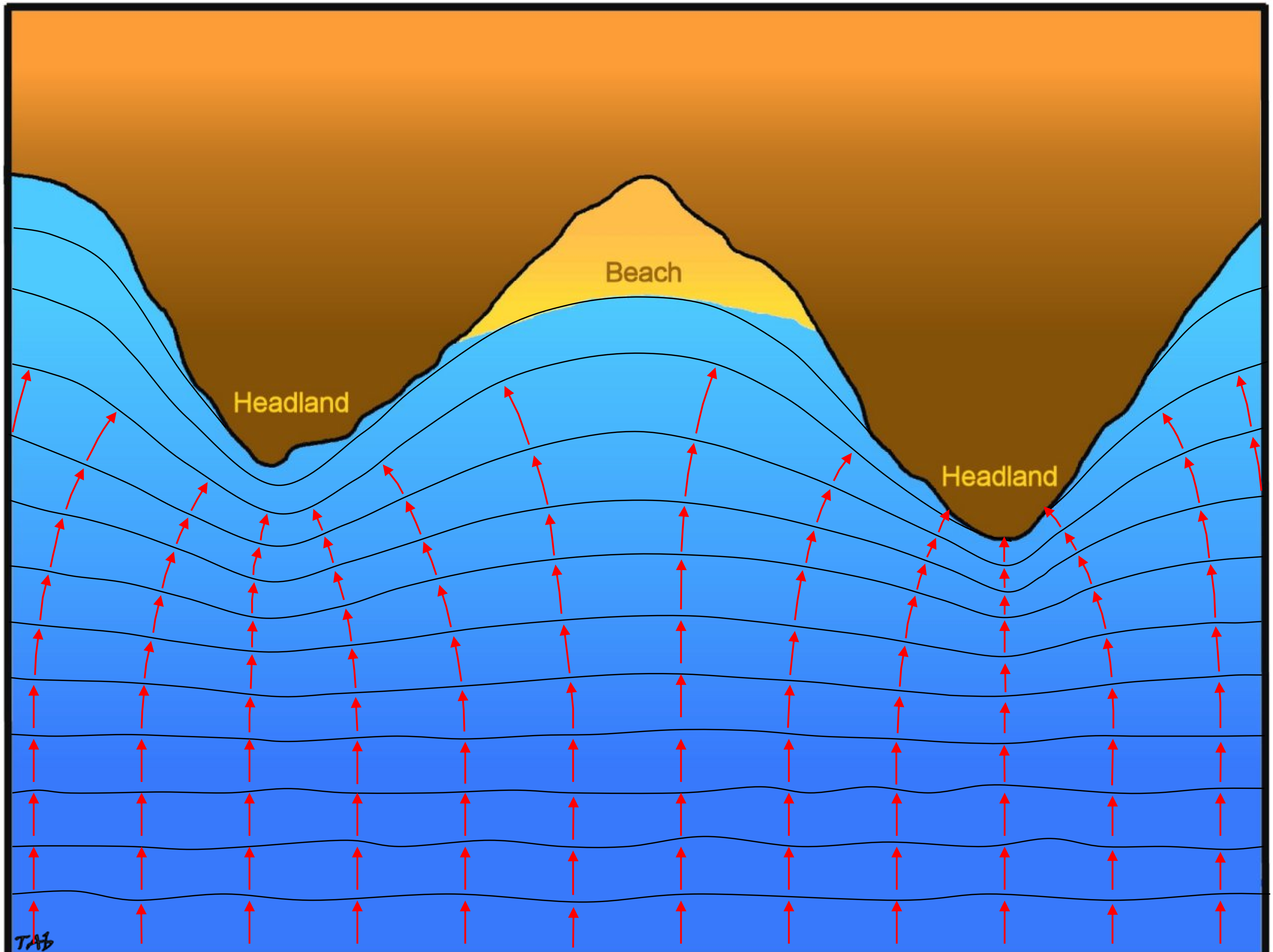

Beach
Headland
Headland
TAB

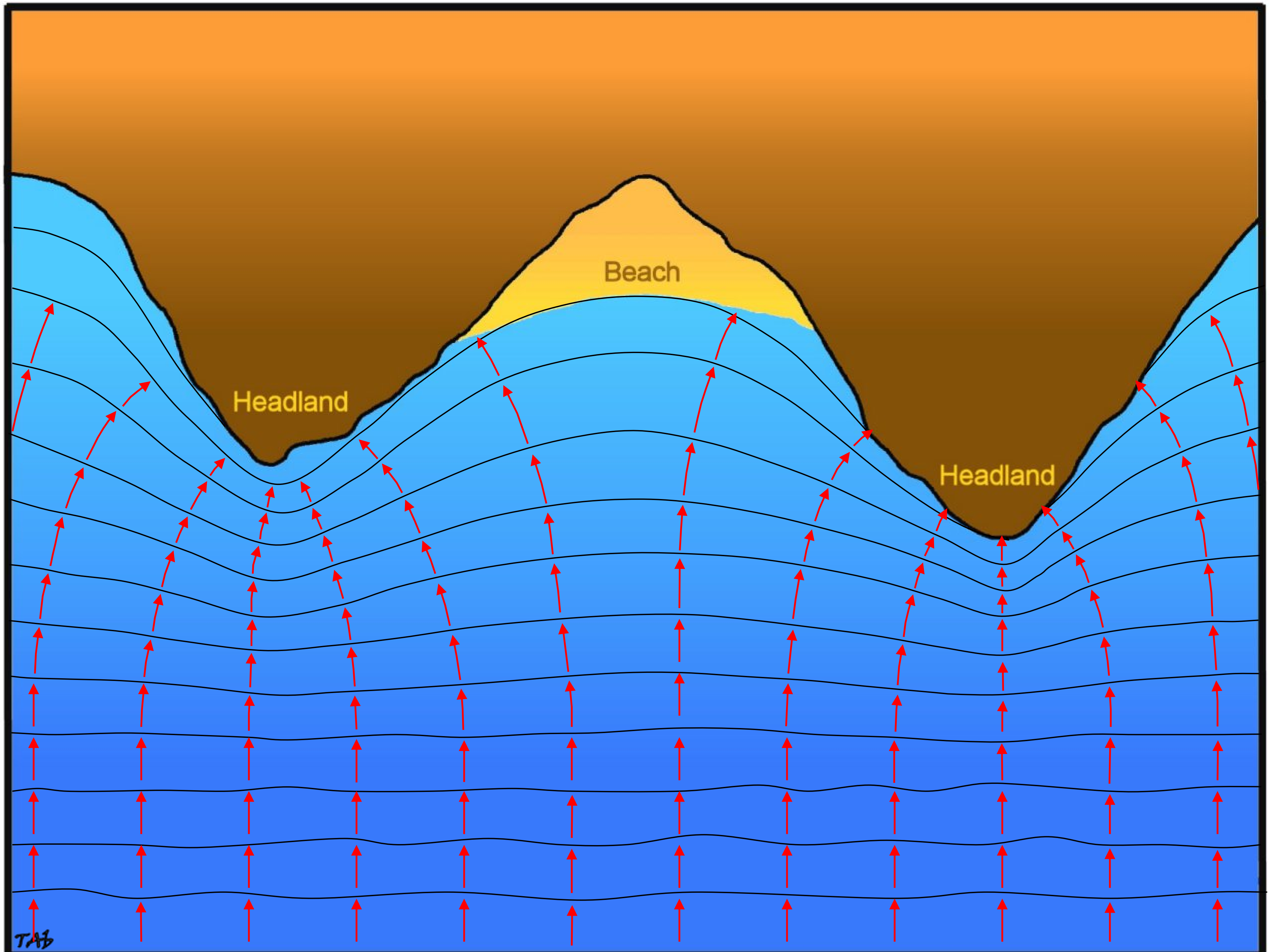

Beach
Headland
Headland
TAB

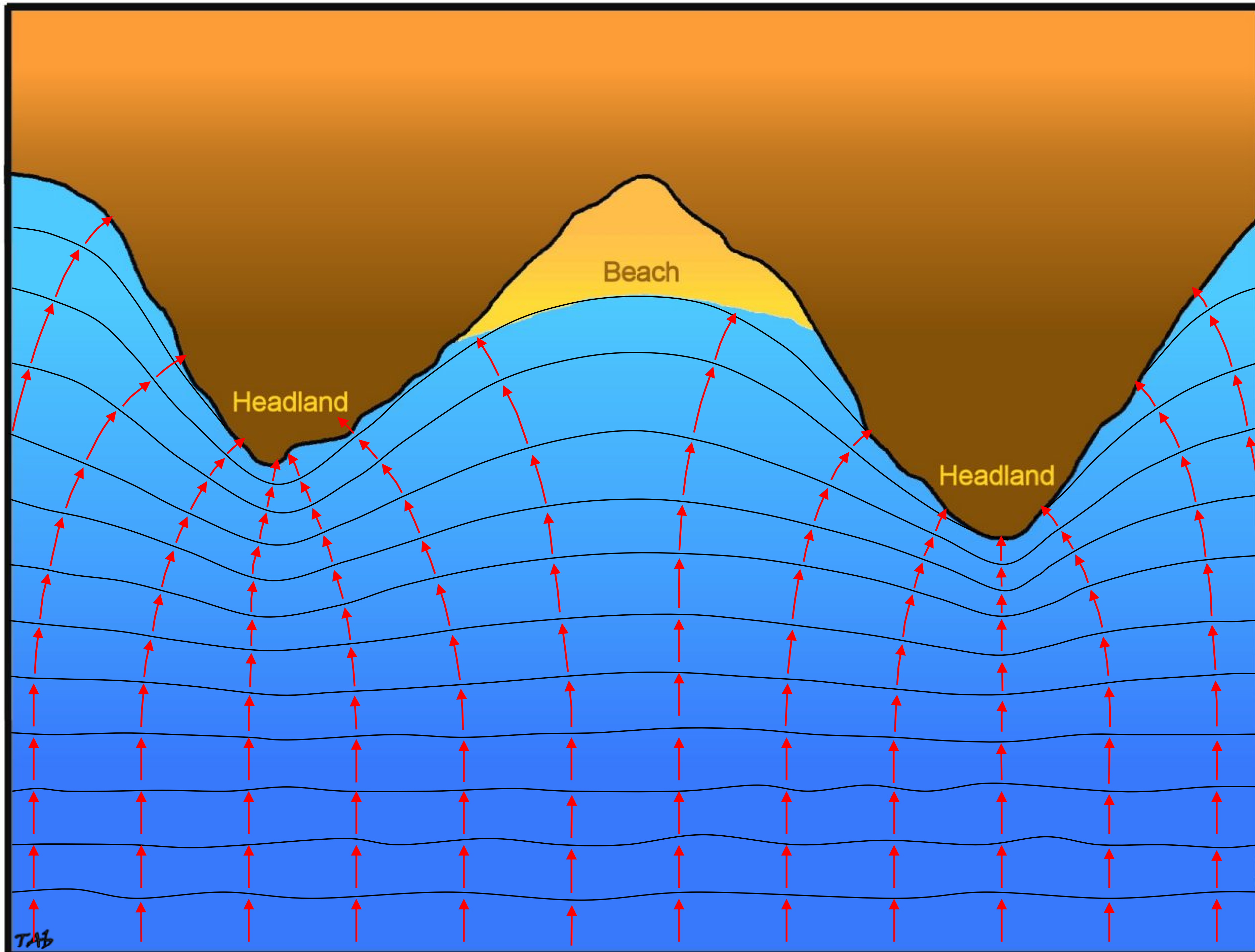

Beach
Headland
Headland
TAB

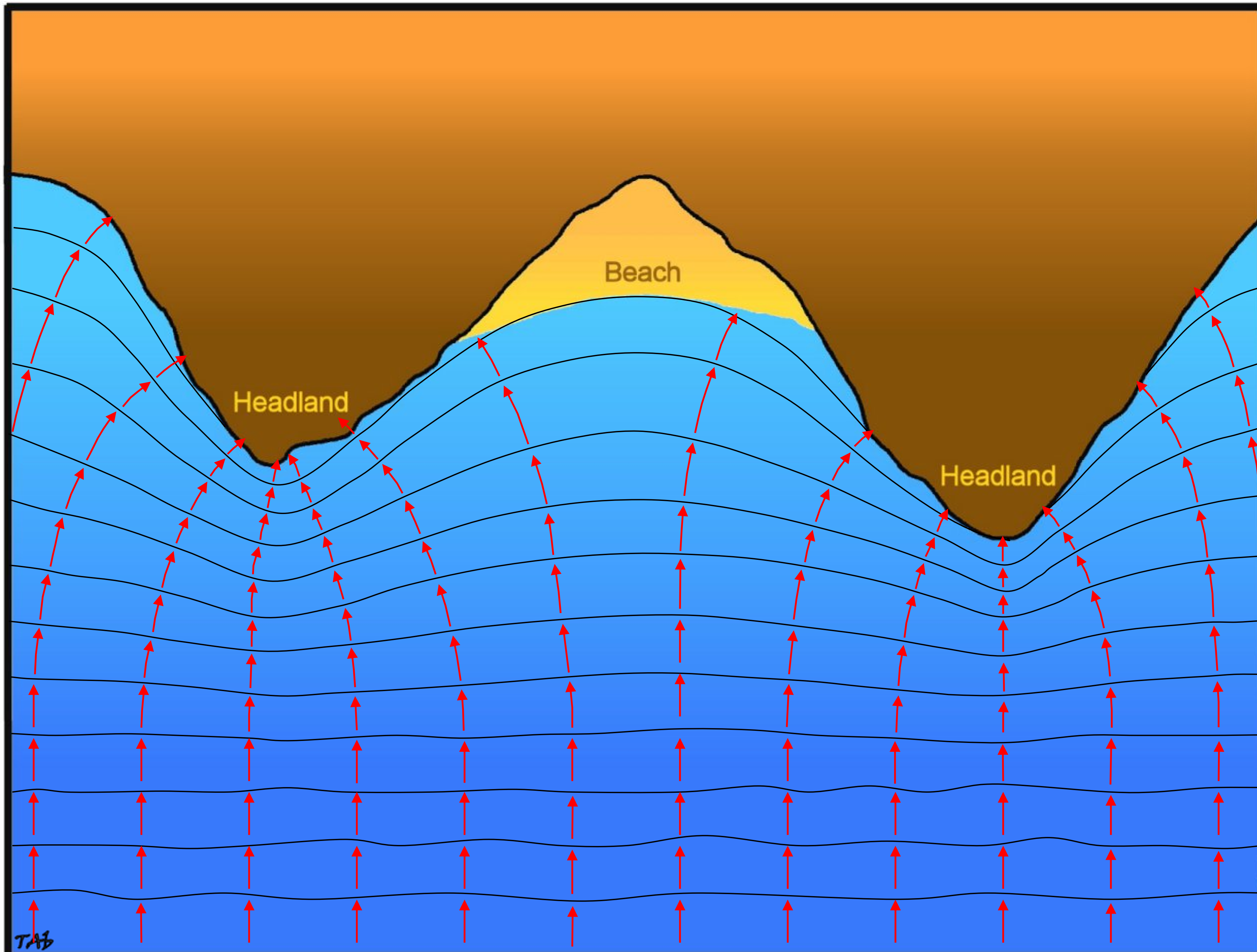


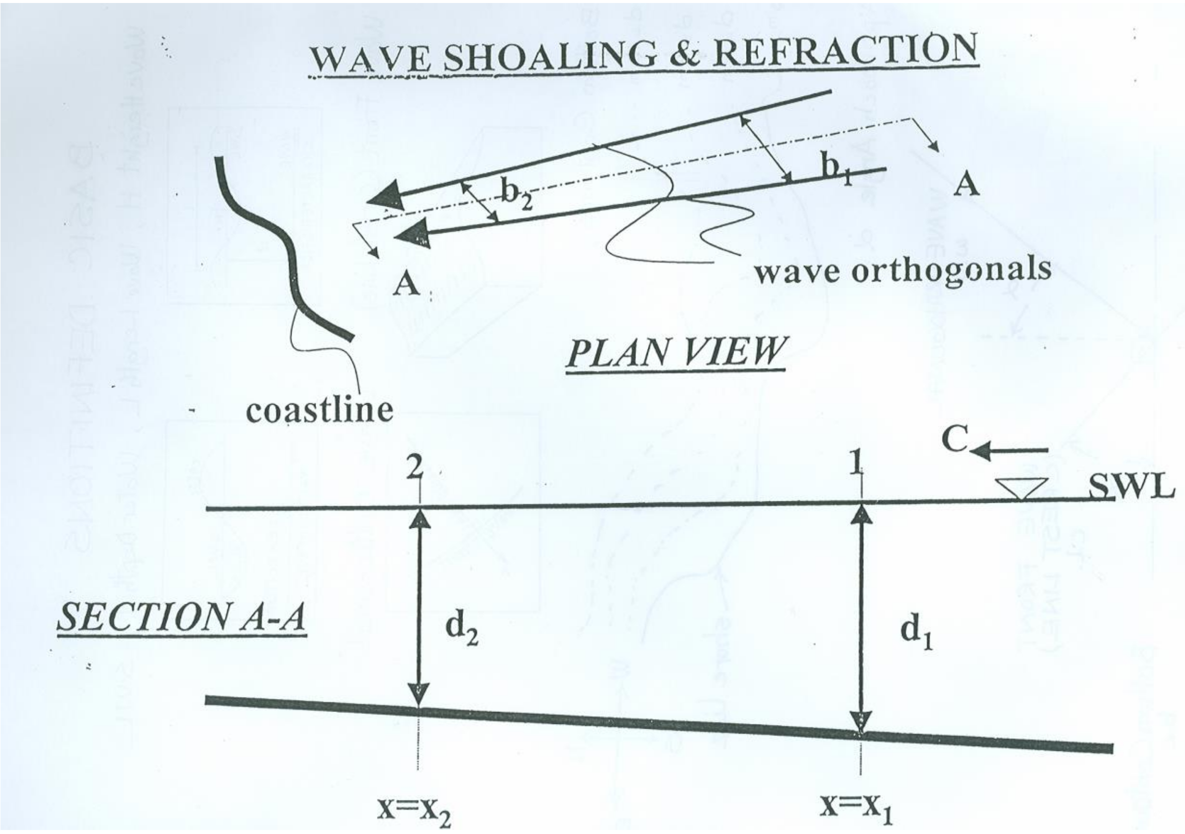
Beach
Headland
Headland
TAB


Beach
Headland
Headland
TAB


Beach
Headland
Headland
TAB


Beach
Headland
Headland
TAB


Beach
Headland
Headland
TAB



Known: T and H_1

Asked: H_2

$$\frac{H_1}{H_o} = (KsKr)_1$$

$$\frac{H_2}{H_o} = (KsKr)_2$$

$$\frac{H_1}{H2} = \frac{(KsKr)_1}{(KsKr)2}$$

EXAMPLE PROBLEM

Simple Refraction-Shoaling Calculation

A wave in deep water has the following characteristics: H_0=3.0 m, T=8.0 sec and α_0=30° . Calculate H and α in 10m and 2m of water depth.

Answer:

$L_0 = gT^2/2\pi = 100\text{m}$

For 10m depth:

d/L_0 = 0.10 and from wave table,

d/L = 0.14, Tanh(kd) = 0.71 and n = 0.81

n=(1+G)/2

→ K_s = 0.93

→ α = 20.9°

$$\frac{\sin\alpha}{\sin\alpha_0} = \frac{C}{C_0} = \tanh\frac{2\pi d}{L}$$

→ K_r = 0.96

$$K_r = \sqrt{\frac{b_0}{b}} = \sqrt{\frac{\cos\alpha_0}{\cos\alpha}}$$

$$\frac{H}{H_0} = \sqrt{\frac{n_0}{n}\frac{C_0}{C}}\sqrt{\frac{b_0}{b}} = \sqrt{\frac{1}{2}\frac{1}{n}\frac{1}{\tanh kd}}\sqrt{\frac{b_0}{b}} = K_s K_r$$

→ **H = 2.70 m**

Gravity Wave Table

d/L_o	$\tanh 2\pi d/L$	d/L	$2\pi d/L$	$\sinh 2\pi d/L$	$\cosh 2\pi d/L$	$\frac{4\pi d/L}{\sinh 4\pi d/L}$ (G)	K_s (Ks)
0.00	0.000	0.0000	0.000	0.000	1.00	1.000	—
01	248	0403	253	256	03	0.958	1.44
02	347	0576	362	370	07	918	23
03	420	0714	448	463	10	877	12
04	480	0833	523	548	14	839	06
0.05	0.531	0.0942	0.592	0.627	1.18	0.800	1.02
06	575	104	655	703	22	763	0.993
07	614	114	716	778	27	725	971
08	649	123	774	854	31	690	955
09	681	132	831	0.930	36	654	942
0.10	0.709	0.141	0.886	1.01	1.42	0.621	0.933
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
12	759	158	0.994	16	54	555	920
13	780	166	1.05	25	60	524	917
14	800	175	10	33	67	494	915
0.15	0.818	0.183	1.15	1.42	1.74	0.465	0.913
16	835	192	20	52	82	437	913
17	850	200	26	61	90	410	913
18	864	208	31	72	1.99	384	914
19	877	217	36	82	2.08	359	916
0.20	0.888	0.225	1.41	1.94	2.18	0.335	0.918
21	899	234	47	2.06	28	313	920
22	909	242	52	18	40	291	923
23	918	251	58	31	52	270	926
24	926	259	63	45	65	251	929
0.25	0.933	0.268	1.68	2.60	2.78	0.233	0.932
26	940	277	74	76	2.93	215	936
27	946	285	79	2.92	3.09	199	939
28	952	294	85	3.10	25	183	942
29	957	303	90	28	43	169	946
0.30	0.961	0.312	1.96	3.48	3.62	0.155	0.949
31	965	321	2.02	69	3.83	143	952
32	969	330	08	3.92	4.04	131	955
33	972	339	13	4.16	28	120	958
34	975	349	19	41	52	110	961
0.35	0.978	0.358	2.25	4.68	4.79	0.100	0.964
36	980	367	31	4.97	5.07	091	967
37	982	377	37	5.28	37	083	969
38	984	386	42	61	5.70	076	972
39	986	396	48	5.96	6.04	069	974
0.40	0.988	0.405	2.54	6.33	6.41	0.063	0.976
41	989	414	60	6.72	6.80	057	978
42	990	424	66	7.15	7.22	052	980
43	991	434	72	7.60	7.66	047	982
44	992	443	79	8.08	8.14	042	983
0.45	0.993	0.453	2.85	8.58	8.64	0.038	0.985
46	994	463	91	9.13	9.19	.035	986
47	995	472	2.97	9.71	9.76	031	987
48	995	482	3.03	10.3	10.4	028	988
49	996	492	09	11.0	11.0	026	990
0.50	0.996	0.502	3.15	11.7	11.7	0.023	0.990

In 2 m of water depth, similar computations yield: d/L_0=0.02, d/L=0.058, tanh $(2\pi d/L)$=0.36 n=0.96, K_s=1.23, α=10.0°, K_r=0.94 and H = 3.5 m.

HOMEWORK

1) Waves of T = 9 sec., and H = 2.2 m. are measured at a depth of 25 m. with an approach angle α_{25} = 30° at this depth. Sea bottom has a uniform slope. Find deep water wave height H_o, and deep water approach angle α_o.

2) A wave of period T = 8 sec. is approaching from deep water towards shore on straight and parallel bottom contours. The angle of incidence of the wave at d = 15 m water depth is α_{15} = 15°. If the wave height at d = 10 m. is H_{10} = 2 m. Calculate;

a) Wave height at d = 15 m.

b) The wave height at deep water.