

ON FINITE DIMENSIONAL ALGEBRAS WHICH ARE SUMS OF TWO SUBALGEBRAS

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ABSTRACT. Rings (algebras) R which are sums of two subrings (subalgebras) R_1 and R_2 , i.e., of the form $R = R_1 + R_2$, have been studied by many authors (e.g., Kegel [4]-[6], Beidar and Mikhalev [1], Bokut [2], Ferrero and Puczyłowski [3], Kepczyk and Puczyłowski [10]-[11]-[12]-[13], Kepczyk [7]-[8]-[12], Felzenszwalb-Giambruno-Leal [5], Salva [14]-[15], Smoktunowicz [16]-[17]). The problem of classifying all such rings (algebras) seems to be far from a complete solution. We classify associative algebras of dimension 2 or 3 over an arbitrary field that are a sum of two subalgebras.

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