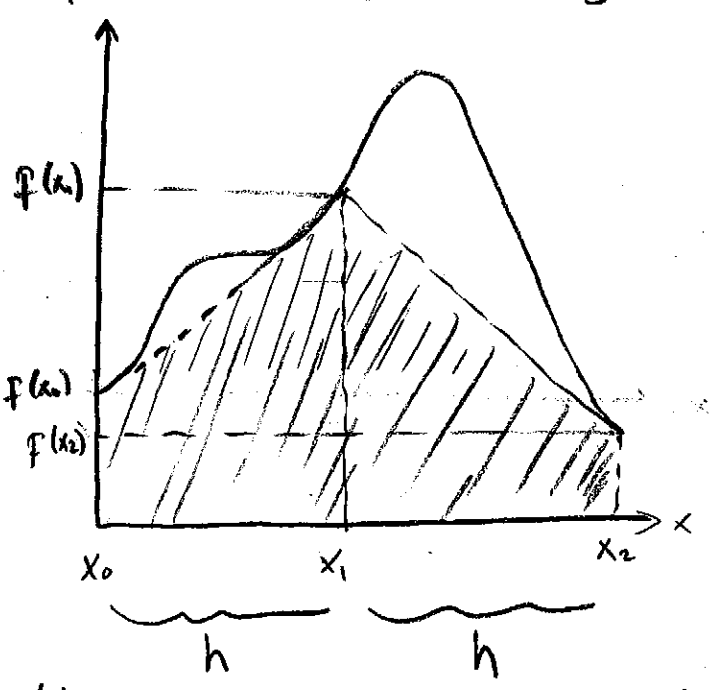
Sayısal İntegral için Trapez Kuralı

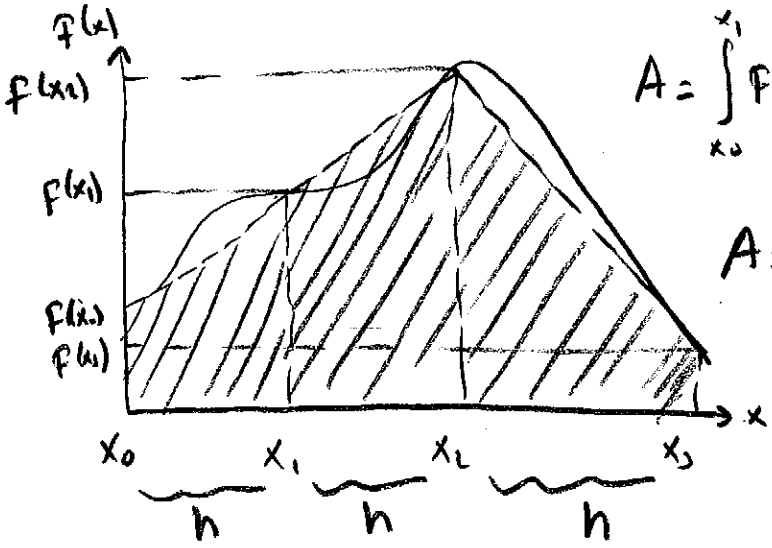


$n = 2$

$h = \frac{x_2 - x_0}{n} = \frac{b-a}{n}$

$$A = \int_{x_0}^{x_1} f(x)dx + \int_{x_1}^{x_2} f(x)dx$$

$$A = \frac{f(x_0)+f(x_1)}{2} \cdot \overbrace{(x_0 - x_1)}^{h} + \frac{f(x_2)+f(x_1)}{2} \cdot \overbrace{(x_2 - x_1)}^{h}$$



$$A = \int_{x_0}^{x_1} f(x)dx + \int_{x_1}^{x_2} f(x)dx + \int_{x_2}^{x_3} f(x)dx$$

$$A = \frac{f(x_0)+f(x_1)}{2} \cdot h + \frac{f(x_2)+f(x_1)}{2} \cdot h + \frac{f(x_3)+f(x_2)}{2} \cdot h$$

$$A = \frac{f(x_0)+f(x_1)}{2} \cdot h + \frac{f(x_2)+f(x_1)}{2} \cdot h + \dots h \cdot \frac{f(x_n)+f(x_{n-1})}{2} \cdot h$$

$$A = \frac{h}{2}\left[f(x_0) + f(x_n) + 2\sum_{i=1}^{n-1} f(x_i) \right]$$

Trapez rule için örnek;

$$V(t) = \frac{gm}{c}\left(1 - e^{-(c/m)t}\right)$$ parasütle düşen birim-cül hız denklemi

g = yerçekimi ivmesi = 9.81 m/sec²

m = parasütçünün kütlesi = 68.1 kg

c = katsayı = 12.5 kg/s

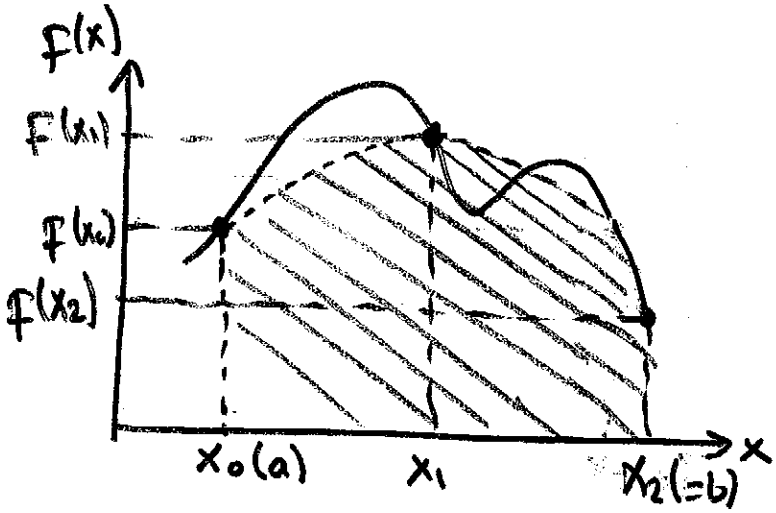
Bir parasütçünün t zaman sonra ne kadar yol aldığı

$$d = \int_0^t V(t)dt \quad \Rightarrow d = \frac{gm}{c}\int_0^t \left(1 - e^{-(c/m)t}\right)dt$$

$t = 10$ sn olsun. Analitik değeri $\Rightarrow d = 289.43515$

$n = 10$

Simpson's 1/3 Rule

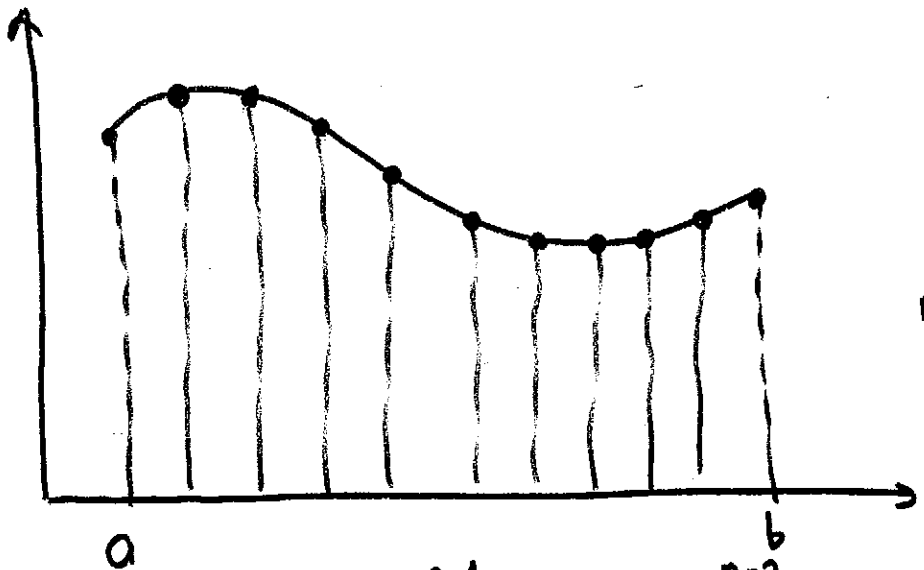


$$A \cong \frac{h}{3}\left[f(x_0)+4f(x_1)+f(x_2)\right]$$

$$h = \frac{b-a}{2}$$

$$A \cong (b-a)\ \frac{f(x_0)+4f(x_1)+f(x_2)}{6}$$

Multiple Application of Simpson's 1/3 Rule (dilim sayısı çift sayı olacak)



n: dilim sayısı

$$A = (b-a)\cdot \frac{f(x_0)+4\sum_{i=1,3,5}^{n-1} f(x_i)+2\sum_{j=2,4,6}^{n-2} f(x_j)+f(x_n)}{3n}$$

Örnek: $f(x) = 0.2 + 25x - 200x^2 + 675x^3 - 900x^4 + 400x^5$

$a = 0 \quad b = 0.8$

Çözüm: $n = 4 \quad (h = 0.2)$

$f(0) = 0.2$ $\quad f(0.4) = 2.456$ $\quad f(0.8) = 0.232$

$f(0.2) = 1.288$ $\quad f(0.6) = 3.464$

$$A = 0.8 \; \frac{0.2 + 4(1.288 + 3.464) + 2.(2.456) + 0.232}{12} = 1.623467$$

Analitik değeri $= 1.640533$

$\text{Hata} = 1.640533 - 1.623467 = 0.017067 \quad \varepsilon_t = \frac{0.017067}{1.640533}$

$\varepsilon_t = \%1.04$