$$\frac{dy}{dx} = f(x, y)$$

EULER'S METHOD

$y_{i+1} = y_i + f(x_i, y_i)h$ h=step size

HEUN'S METHOD

Predictor $y^o_{i+1} = y_i + f(x_i, y_i)h$

Corrector $y_{i+1} = y_i + \frac{f(x_i, y_i) + f(x_{i+1}, y^o_{i+1})}{2} h$

THE MIDPOINT (OR IMPROVED POLYGON) METHOD

Predictor $y_{i+1/2} = y_i + f(x_i, y_i)\frac{h}{2}$

Corrector $y_{i+1} = y_i + f(x_{i+1/2}, y_{i+1/2})h$

RUNGE-KUTTA METHODS

Second-Order Runge-Kutta Methods

Ralston's Method

$$y_{i+1} = y_i + \left(\frac{1}{3}k_1 + \frac{2}{3}k_2\right)h$$
$$k_1 = f(x_i, y_i)$$
$$k_2 = f\left(x_i + \frac{3}{4}h, y_i + \frac{3}{4}k_1 h\right)$$

Third-Order Runge-Kutta Methods

$$y_{i+1} = y_i + \frac{1}{6}(k_1 + 4k_2 + k_3)h$$
$$k_1 = f(x_i, y_i)$$
$$k_2 = f\left(x_i + \frac{1}{2}h, y_i + \frac{1}{2}k_1 h\right)$$
$$k_3 = f(x_i + h, y_i - k_1 h + 2k_2 h)$$

Fourth-Order Runge-Kutta Methods

$$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)h$$
$$k_1 = f(x_i, y_i)$$
$$k_2 = f\left(x_i + \frac{1}{2}h, y_i + \frac{1}{2}k_1 h\right)$$
$$k_3 = f\left(x_i + \frac{1}{2}h, y_i + \frac{1}{2}k_2 h\right)$$
$$k_4 = f(x_i + h, y_i + k_3 h)$$