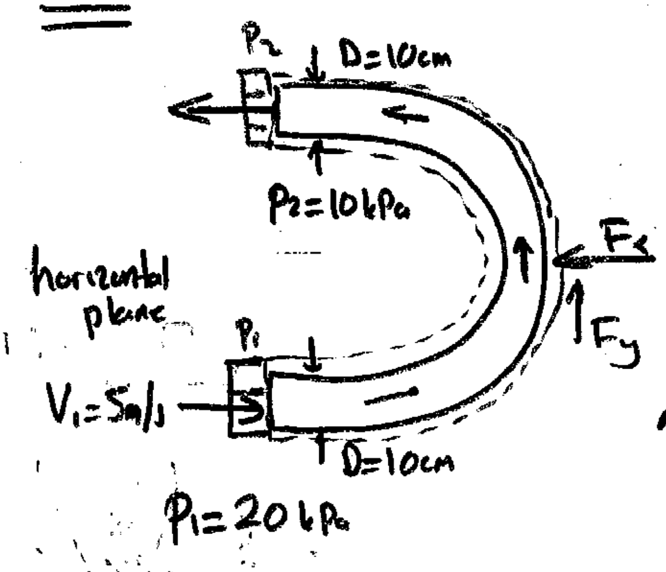
Ex:



Calculate the horizontal (x and y) components of the anchoring force required to hold the bend in place

Momentum eq in x direction

$$-F_x + p_1A_1 + p_2A_2 = -\rho \overset{-}{Q} \overset{+}{V_1} - \rho \overset{+}{Q} \overset{-}{V_2}$$

$$F_x = p_1A_1 + p_2A_2 + \rho Q(V_1+V_2)$$

$$F_y = 0$$

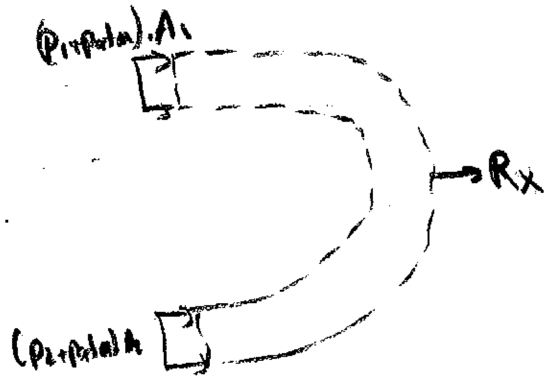
$$F_x = 20000 . \frac{\pi \times 0.1^2}{4} + 10000 . \frac{\pi \times 0.1^2}{4} + 1000 \times 5 \times \frac{\pi \times 0.1^2}{4}(5+5)$$

$$F_x = 157.08 + 78.5 + 392.7 = 628.28 \text{ N}$$

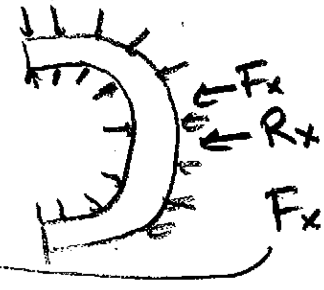
The anchoring force for the pipe bend is independent of the atmospheric pressure. However the force that the bend puts on the fluid inside of it depends on the atmosphere pressure.

$$R_x + (p_1+p_{atm})A_1 + (p_2+p_{atm})A_2 = -\rho Q(V_1+V_2)$$

$$R_x = -(p_1+p_{atm})A_1 - (p_2+p_{atm})A_2 - \rho Q(V_1+V_2)$$



$$F_x + R_x + p_{atm}(A_1+A_2) = 0$$

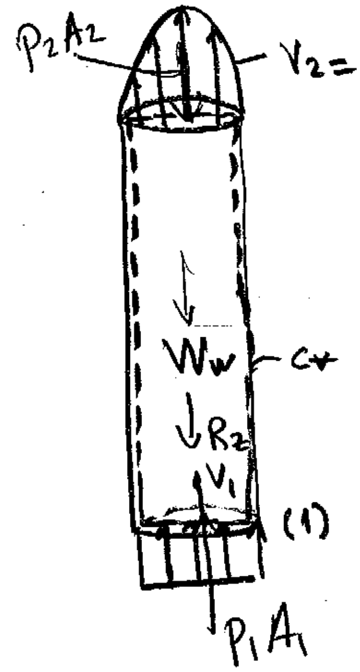


$$F_x = -R_x - p_{atm}(A_1+A_2)$$

$$F_x = (p_1+p_{atm})A_1 + (p_2+p_{atm})A_2 + \rho Q(V_1+V_2) - p_{atm}(A_1+A_2)$$

$$F_x = p_1A_1 + p_2A_2 + \rho Q(V_1+V_2)$$

Ex:



R_z: is the resultant force of the wetted pipe wall on the fluid

Momentum eq. in z direction

$$p_1A_1 - p_2A_2 - W_w - R_z = -\rho \bar{Q} \vec{V}_1 + \rho \int V_2 V_2 dA$$

$$\rho \int V_2 V_2 dA = \rho \int_0^{R_2} \left[2V_1\left(1-\left(\frac{r}{R}\right)^2\right)\right]^2 \cdot 2\pi r dr$$

$$= \rho\, 4V_1^2 \int_0^R \left(1 - 2\frac{r^2}{R^2} + \frac{r^4}{R^4}\right) 2\pi r dr$$

$$= \rho\, 8\pi V_1^2 \int_0^R \left(r - \frac{2r^3}{R^2} + \frac{r^5}{R^4}\right) dr$$

$$= \rho\, 8\pi V_1^2 \left[\frac{R^2}{2} - \frac{2R^4}{4R^2} + \frac{R^6}{6R^4}\right] = \rho\, 8\pi V_1^2 \frac{R^2}{6} = \frac{4}{3}\pi \rho V_1^2 R^2$$

$$p_1A_1 - p_2A_2 - W_w - R_z = -\rho V_1 V_1 \pi R_1^2 + \frac{4}{3}\pi \rho V_1^2 R^2 \quad (A_1 = A_2)$$

$$p_1 - p_2 = \frac{W_w}{A} + \frac{R_z}{A} + \frac{\rho V_1^2}{3}$$

For steady flows

$$\sum \vec{F} = \int_{CS} \rho \vec{V} (\vec{V} \cdot \vec{n}) \, dA$$

$$\sum \vec{F}_i = \sum_{i=1}^{N} \rho V_i \dot{m}_i = \sum_{i=1}^{N} \rho_i Q_i V_i$$

$\sum F = F_{body} + F_{surface}$

F_{body}: Weight

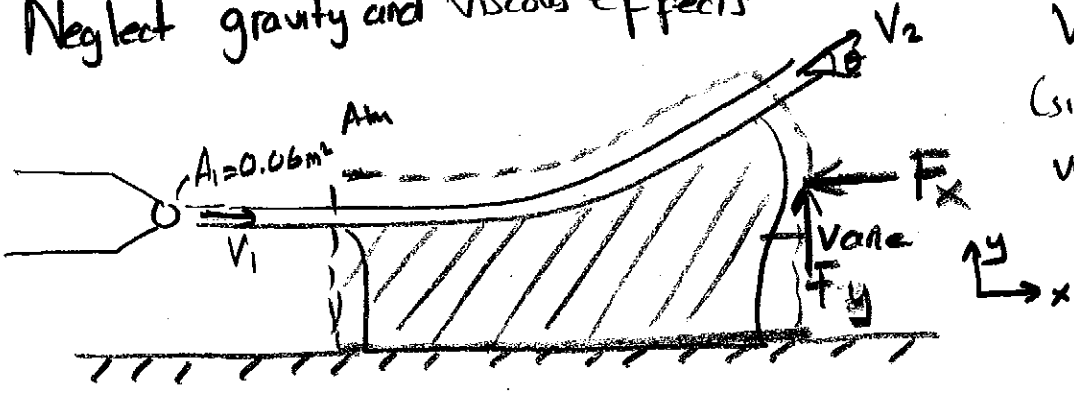
$F_{surface}$: Friction, pressure
Reaction forces

Example: A horizontal jet of water exits a nozzle with a uniform speed of $V_1 = 10$ m/s, strikes a vane and is turned an angle θ. Determine the anchoring force needed to hold the vane stationary. Neglect gravity and viscous effects

$\theta = 30°$

$V_1 = V_2$

(since the gravity and viscous effects are neglected)



$$Q_1 = Q_2 = 10 \times \frac{\pi \cdot 0.06^2}{4} = 0.0283 \, m^3/s$$

Momentum eq in x direction

$$-F_x = -\rho \overset{(-)}{Q} \overset{(+)}{V_1} + \rho \overset{(+)}{Q} \overset{(+)}{V_2 \cos\theta}$$

$$-F_x = -1000 \times 0.0283 \times 10 + 1000 \times 0.0283 \times 10 \cdot \cos 30$$

$$-F_x = -283 + 245.09$$

$$F_x = 37.9 \, N$$

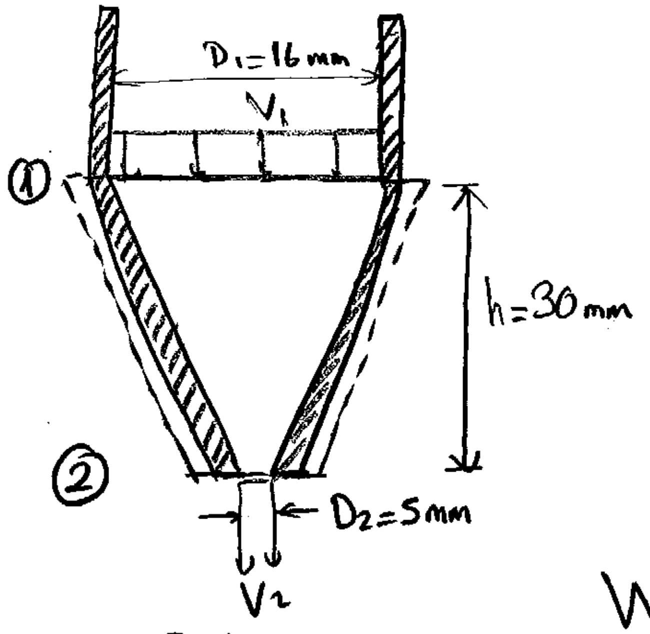
Momentum equation in y direction

$$F_y = 1000 \times \overset{+}{0.0283} \times 10 \cdot \overset{+}{\sin 30}$$

$$F_y = 141.5 \, N$$

$$F = \sqrt{37.9^2 + 141.5^2} = 146.5 \, N$$

Ex: Determine the anchoring force required to hold in place a conical nozzle attached to the end of a laboratory sink faucet when the water flowrate is 0.6 lt/s. The nozzle mass is 0.1 kg. The nozzle inlet and exit diameters are 16 mm and 5 mm, respectively. The nozzle axis is vertical and the axial distance between sections (1) and (2) is 30 mm. The gage pressure at section (1) is 464 kPa.



$$V_1 = \frac{Q}{A_1} = \frac{0.6\times10^{-3}}{\frac{\pi \cdot 0.016^2}{4}} = 2.98 \text{ m/s}$$

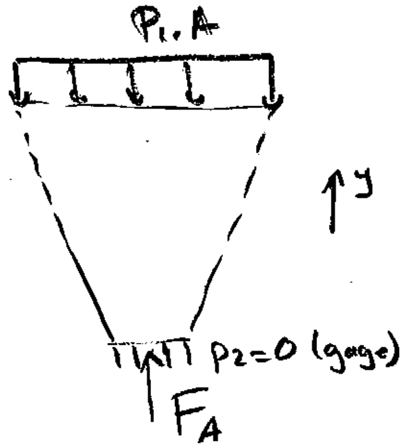
$$V_2 = \frac{Q}{A_2} = \frac{0.6\times10^{-3}}{\frac{\pi \times 0.005^2}{4}} = 30.6 \text{ m/s}$$

$$W_w = \rho \forall_w . g$$

$$W_w = \rho \frac{1}{12}\pi h \left(D_1^2 + D_2^2 + D_1 D_2\right) . g$$

$$W_w = 0.0278 \text{ N}$$

$$W_n = 0.1 \times 9.81 = 0.981 \text{ N}$$



Momentum equation in y direction

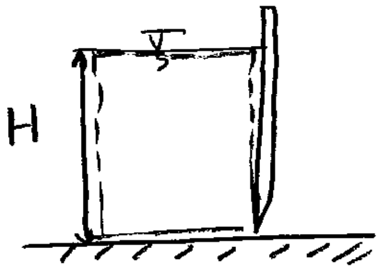
$$F_A - p_1 . A_1 - W_w - W_n = +\rho \bar{Q} \bar{V}_1 - \rho \bar{Q} \bar{V}_2$$

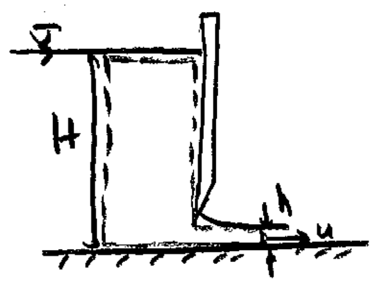
$$F_A - 464000 \cdot \frac{\pi \cdot 0.016^2}{4} - 0.0278 - 0.981 = 1000 \times 0.6\times10^{-3} \times 2.98 - 1000 \times 0.6\times10^{-3} \times 30.6$$

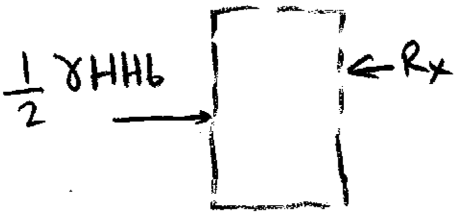
$$F_A - 93.29 - 0.0279 - 0.981 = 1.788 - 18.36$$

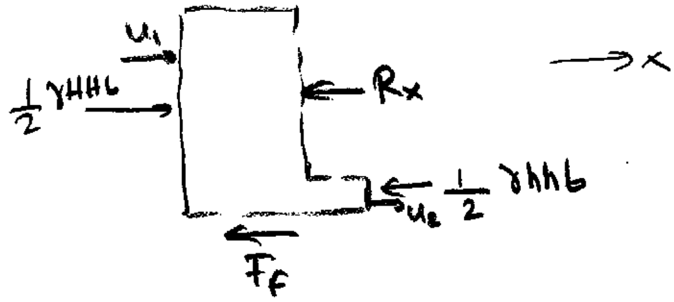
$$F_A = 77.7 \text{ N}$$

Ex: A sluice gate across a channel of width b is shown in the closed and open positions. Is the anchoring force required to hold the gate in place larger when the gate is closed or when it is open?









(A) $R_x = \frac{1}{2}\gamma H^2 b$

$$-R_x - F_f + \frac{1}{2}\gamma H^2 b - \frac{1}{2}\gamma h^2 b = -\rho u_1 b u_1 H + \rho u_2 h b u_2$$

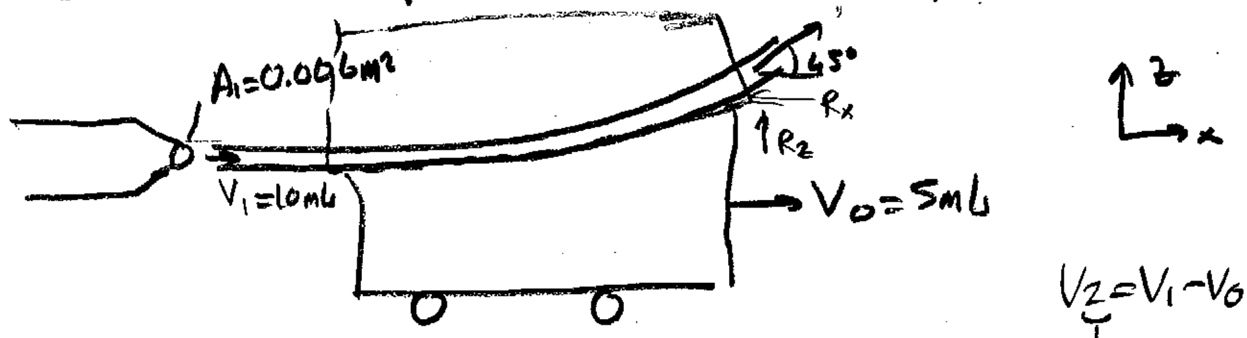
$$-R_x - F_f + \frac{1}{2}\gamma H^2 b - \frac{1}{2}\gamma h^2 b = -\rho u_1^2 b H + \rho u_2^2 b h$$

If $H >> h$ ise $u_1 << u_2$

(B) $R_x = \frac{1}{2}\gamma H^2 b - \frac{1}{2}\gamma h^2 b - F_f - \rho u_2^2 h b$

$R_{xA} > R_{xB}$

Ex: Determine the magnitude and direction of the force, F, exerted by the stream of water on the vane surface.



$V_2 = V_1 - V_0$

$$-R_x = -\rho (V_1 - V_0) A_1 \cdot (V_1 - V_0) + \rho (V_1 - V_0) A_1 \cdot V_2 \cos 45$$

$$-R_x = -\rho (10-5) \times 0.006 \times (10-5) + 1000(10-5) \times 0.006 \times 5 \cdot \cos 45$$

$$-R_x = -150 + 106.07$$

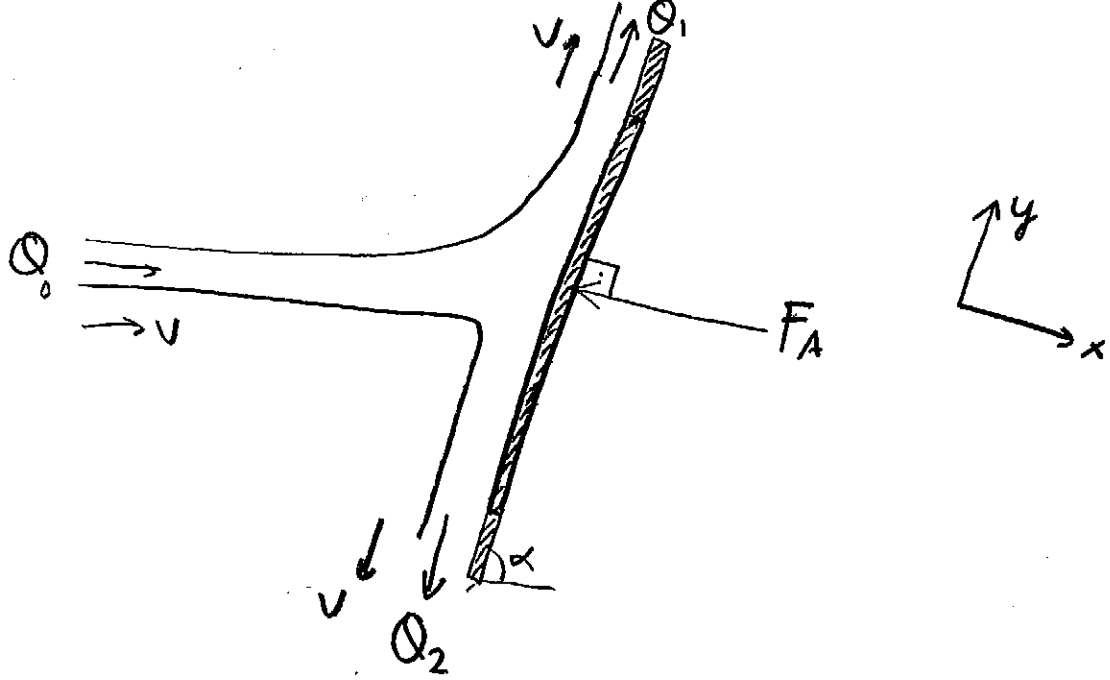
$$-R_x = -43.93$$

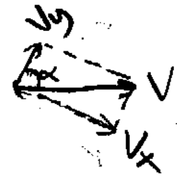
$$R_x = 43.93\,N$$

$$R_z = \rho (V_1 - V_0) A_1 V_2 \sin 45$$

$$R_z = 1000(5) \times 0.006 \cdot 5 \cdot \sin 45$$

$$R_z = 106.1\,N$$





$$Q_0 = Q_1 + Q_2$$

Momentum equation in y direction

$$0 = -\rho Q_0 V \cdot \cos\alpha + \rho Q_1 V - \rho Q_2 V$$

$$Q_0 \cdot \cos\alpha + Q_2 = Q_1$$

$$(Q_1 + Q_2) \cdot \cos\alpha + Q_2 = Q_1$$

$$Q_1 (1 - \cos\alpha) = Q_2 (1 + \cos\alpha) \Rightarrow \frac{Q_1}{Q_2} = \frac{1+\cos\alpha}{1-\cos\alpha}$$

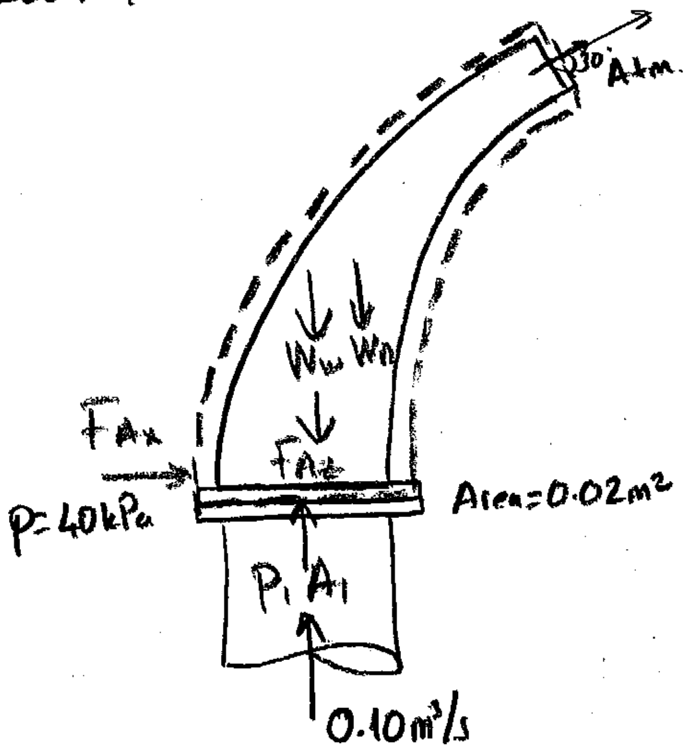
Momentum equation in x direction

$$-F_A = -\rho Q_0 V \cdot \sin\alpha$$

$$F_A = \rho Q_0 V \sin\alpha$$

9

A nozzle is attached to a vertical pipe and discharges water into atmosphere. When the discharge is 0.1 m^3/s, the gage pressure at the flange is 40 kPa. Determine the vertical and horizontal components of the anchoring force required to hold the nozzle in place. The nozzle has a weight of 200 N, and the volume of water in the nozzle is 0.012 m^3.



$$Q_1 = Q_2$$

$$V_1 = \frac{Q_1}{A_1} = \frac{0.10}{0.02} = 5\,m/s$$

$$V_2 = \frac{Q_2}{A_2} = \frac{0.10}{0.01} = 10\,m/s$$

$$P_1 A_1 - F_{Az} - W_w - W_n = -\rho Q V_1 + \rho Q V_2 . \sin 30$$

$$F_{Az} = 40000 \times 0.02 - 200 - 0.012 \times 9810 + 1000 \times 0.1 \times 5 - 1000 \times 0.1 \times 10 \sin 3$$

$$F_{Az} = 482\ N\ (\downarrow)$$

$$F_{Ax} = \rho Q V_{2x} = 1000 \times 0.1 \times 10 . \cos 30 = 866\ N$$