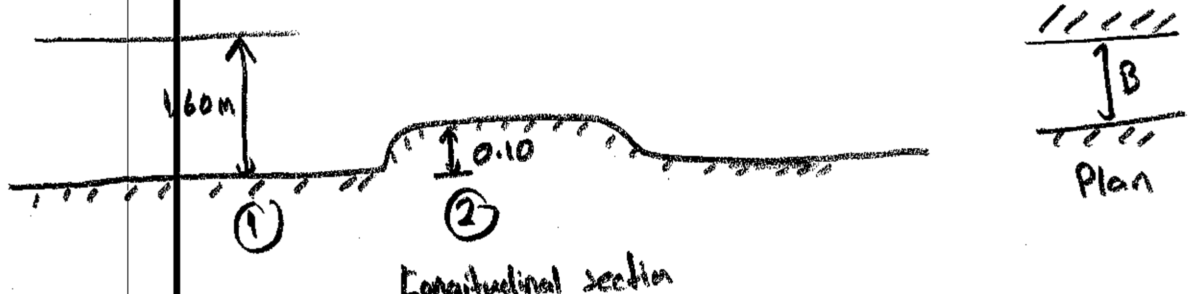
Ex: A rectangular channel has a width of 2.0m and carries a discharge of 4.80 m³/s with a depth of 1.60m. At a certain cross-section a small smooth hump with a flat top and a height 0.10m is proposed to be built. Calculate the likely change in the water surface



$$E_1 = E_2 + \Delta z$$

$$q = \frac{Q}{B} = \frac{4.8}{2} = 2.4\ m^2/s/m \quad ; \quad h_{cr} = \sqrt[3]{\frac{q^2}{g}} = \sqrt[3]{\frac{2.4^2}{9.81}} = 0.837m$$

$1.60m > 0.837m \Longrightarrow$ Subcritical flow

$$E_{min} = 1.5\, h_{cr} = 1.5 \times 0.837 = 1.256m$$

$E_1 - \Delta z > E_{min}$ No chocking

$$E_1 = 1.6 + \frac{1}{19.62} \cdot \left(\frac{2.4}{1.6}\right)^2 = 1.715m$$

$$E_1 - \Delta z = E_2 = (1.715 - 0.1) = h_2 + \frac{1}{19.62} \cdot \left(\frac{2.4}{h_2}\right)^2$$

$$1.615 = h_2 + \frac{0.293}{h_2^2}$$

$$h_2^3 - 1.615 h_2^2 + 0.293 = 0$$

$\boxed{h_2 = 1.48m \text{ (subcritical)}}$

$h_2 = 0.52m$ (supercritical) X

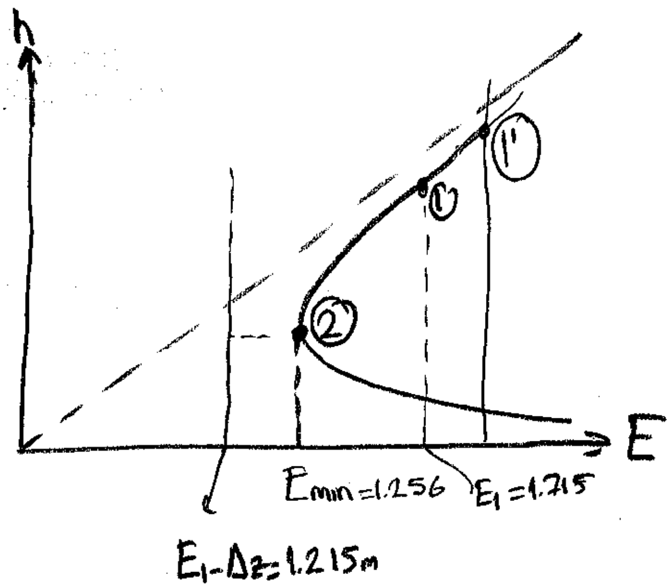
$$\Delta h = 1.60 - (1.48 + 0.1) = 0.02m$$

b) What is the maximum elevation of the hump so that the upstream conditions do not change

$E_2 = E_{min} = 1.256 \Longrightarrow E_1 = E_{min} + \Delta z \Longrightarrow 1.715 = 1.256 + \Delta z_{max}$

$$\Delta z_{max} = 0.459m$$

Ex: In previous example if the height of the hump is 0.50m estimate the water surface elevation on the hump and at a section upstream of the hump?



$\Delta z > \Delta z_{max}$

$E_2 = E_{min_2}$

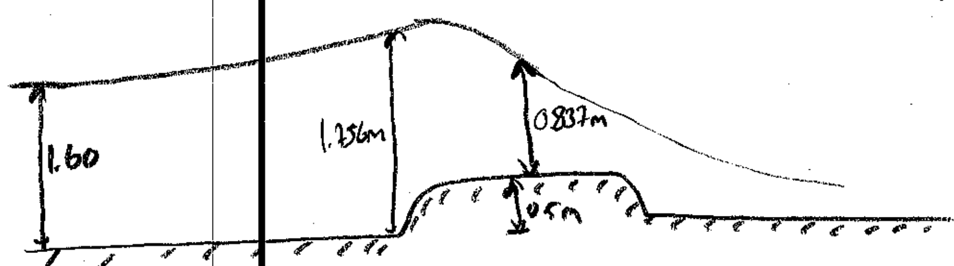
$E_1^* = E_{min_2} + \Delta z$

$E_1^* = 1.256 + 0.5 = 1.756$

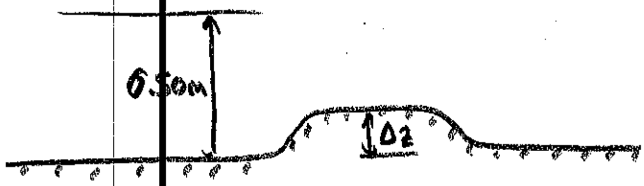
$h_1^* + \left(\frac{2.4}{h_1}\right)^2 \cdot \frac{1}{19.62} = 1.756$

$h_1^* + \frac{0.2936}{(h_1^*)^2} = 1.756$

$(h_1^*)^3 - 1.756(h_1^*)^2 + 0.2936 = 0 \Rightarrow$ $h_1^* = 1.648m$ (subcritical) ✓
$h_1^* = 0.48m$ ✗ (supercritical)



Ex: Rectangular channel 2.50m wide carries 6.0 m³/s of flow at a depth of 0.50 m. Calculate the height of a flat topped hump required to be placed at a section to cause critical flow. The energy loss due to obstruction by the hump can be taken as 0.1 times the upstream velocity head.



$q = \frac{6}{2.5} = 2.4\ m^3/s/m \quad \Rightarrow \quad V_1 = \frac{2.4}{0.5} = 4.8\ m/s \quad E_1 = 0.5 + \frac{4.8^2}{19.62} = 1.674\ m$

$h_{cr} = \sqrt[3]{\frac{2.4^2}{9.81}} = 0.837\ m$

$E_{min} = 1.5 h_{cr} = 1.256\ m$

$$E_1 = E_{min} + \Delta z + h_L$$

$$1.674 = 1.256 + \Delta z + 0.1 \cdot \frac{4.8^2}{19.62}$$

$$\Delta z = 0.3\ m$$

Ex: A 4m wide rectangular channel is carrying 10 m³/s at a depth of 2.5m. There is a step rise of 0.2 m in the channel bottom. Assuming there are no losses at the transition determine the flow depth downstream of bottom step. Does this water surface rise or fall at the step?

$$V_1 = \frac{Q}{A_1} = \frac{10}{4 \times 2.5} = 1 \text{ m/s}$$

$$E_1 = y_1 + \frac{V_1^2}{2g} = 2.5 + \frac{1}{2 \times 9.81} = 2.55\text{m}$$

$$E_2 = E_1 - 0.2 = 2.55 - 0.2 = 2.35\text{m}$$

$$E_2 = y_2 + \frac{Q^2}{2gA_2^2}$$

$$y_2^3 - 2.35y_2^2 + 0.32 = 0$$

y_2 → 2.29 m → since the upstream flow is subcritical
→ 0.405 m
→ −0.345 m

Water level downstream of the transition $= 0.2 + 2.29 = 2.49\text{m}$

Thus the water-surface level drops by $2.5 - 2.49 = 0.01\text{m}$

Q1) A discharge of 16.0 m3/sec flows with a depth of 2.0 m in a rectangular channel 4.0 m wide. At a downstream section the width is reduced to 3.50 m and the channel bed is raised by ΔZ. Analyze the water surface elevations in the transitions when a) ΔZ = 0.20 m and b) ΔZ = 0.35 m.

Solution:

Let the suffixes 1 and 2 refer to the upstream and downstream sections respectively. At the upstream section,

$$V_1 = \frac{16}{4 \times 2} = 2.0\, m/\sec$$

$$F_{r1} = \frac{V_1}{\sqrt{g y_1}} = \frac{2.0}{\sqrt{9.81 \times 2.0}} = 0.45$$

The upstream flow is subcritical and the transition will cause a drop in the water surface elevation.

$$\frac{V_1^2}{2g} = \frac{2.0^2}{19.62} = 0.20m$$

$$E_1 = y_1 + \frac{V_1^2}{2g} = 2.0 + 0.20 = 2.20m$$

For the transition cross-section 2,

$$q_2 = \frac{Q}{B_2} = \frac{16.0}{3.50} = 4.57\, m^2/\sec$$

$$y_{c2} = \left(\frac{q_2^2}{g}\right)^{1/3} = \left(\frac{4.57^2}{9.81}\right)^{1/3} = 1.29m$$

$$E_{c2} = \frac{3}{2} y_{c2} = \frac{3}{2} \times 1.29 = 1.94m$$

c) *When ΔZ = 0.20 m,*

E_2 = Available specific energy at section 2

$$E_2 = E_1 - \Delta Z = 2.20 - 0.20 = 2.00m > E_{c2} = 1.94m$$

Hence the depth $y_2 > y_{c2}$ and the upstream depth will remain unchanged at section 1, y_1.

$$y_2 + \frac{V_2^2}{2g} + \Delta Z = E_1$$

$$y_2 + \frac{4.57^2}{19.62 \times y_2^2} = 2.20 - 0.20 = 2.00m$$

$$y_2 + \frac{1.064}{y_2^2} = 2.00m$$

Solving by trial and error,

$$y_2 = 1.58m$$

Hence when ΔZ = 0.20 m, y_1 = 2.00 m and y_2 = 1.58 m. The drop in water surface is,

$$\Delta h = 2.00 - 1.58 - 0.20 = 0.22m$$

c) *When ΔZ = 0.35 m,*

E_2 = Available specific energy at section 2

$$E_2 = 2.20 - 0.35 = 1.85m < E_{c2} = 1.94m$$

Hence the contraction will be working under the choked conditions. The upstream depth must rise to create a higher to energy. The depth of flow at section 2 will be critical with,

$$y_2 = y_{c2} = 1.29 \text{ m}$$

If the new depth is y_1',

$$y_1' + \frac{Q^2}{2gB_1^2 y_1'^2} = E_{c2} + \Delta Z$$

$$y_1' + \frac{16.0^2}{19.62 \times 4.0^2 \times y_1'^2} = 1.94 + 0.35$$

$$y_1' + \frac{0.815}{y_1'^2} = 2.29m$$

By trial and error,

$$y_1' = 2.10m$$

The upstream depth will therefore rise by 0.10 m due to the choked condition at the constriction. Hence, when ΔZ = 0.35 m,

$$y_1' = 2.10 \text{ m} \quad \text{and } y_2 = y_{c2} = 1.29 \text{ m}$$