

RINGS WITH RESTRICTED INJECTIVITY DOMAINS

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ABSTRACT. Beginning in 1964, with the celebrated theorem of Barbara Osofsky that an associative ring R with identity is semisimple Artinian if and only if every cyclic right module over R is injective, characterizing rings by means of homological properties of their modules has attracted much attention. A number of classes of rings were characterized by homological properties of certain types of their modules. Our work follows in this sound tradition as we investigate the structure of rings in terms of their injective profiles (the class of injectivity domains of their modules). Since every module over a semisimple Artinian ring is injective, semisimple Artinian rings emerge as the simplest type of rings from this perspective. We will be interested then in the second step, those rings which are not semisimple Artinian and have exactly two feasible domains of injectivity: semisimple modules and all modules; these are the so-called *rings without a middle class*. We further the study of these rings by focusing on an interpretation of that property in terms of the lattice of hereditary pretorsion classes over a given ring. For non-semisimple rings, the absence of a middle class is equivalent to the requirement that the class of all semisimple right modules be a coatom in that lattice. Taking advantage of this perspective, we discover new facts and shed light onto others already known with a possibly more direct interpretation without having to refer to an exhaustive analysis of the structure theorems available in the literature. We also discuss the open problem of whether there is a ring with no right middle class which is not right Noetherian.