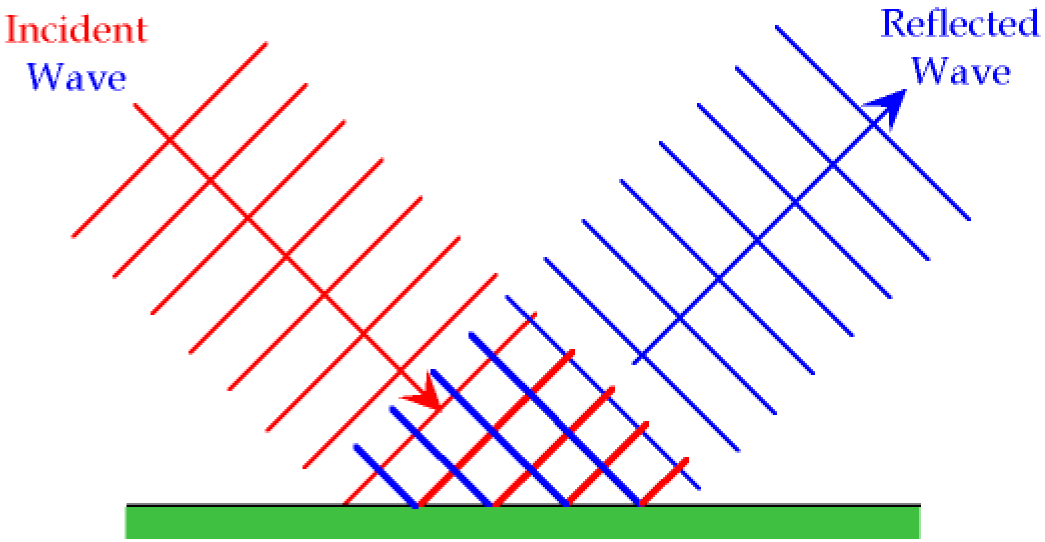
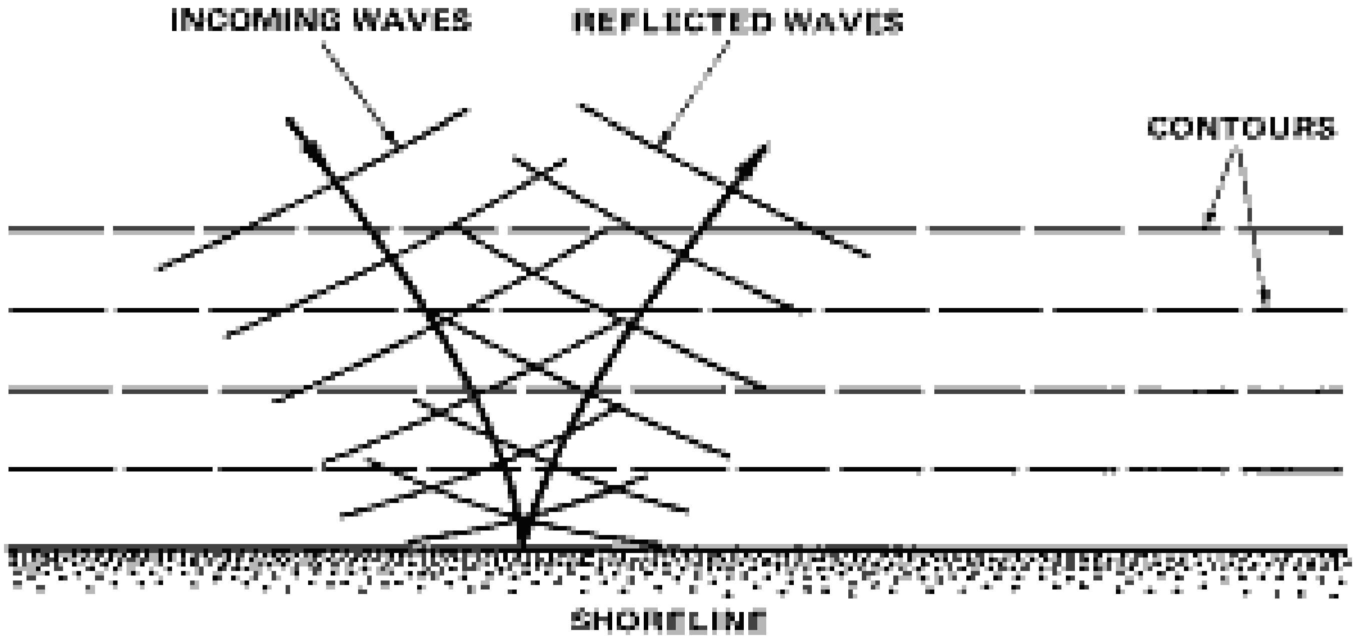
WAVE REFLECTION

Water waves are reflected from hard flat surfaces as shown below

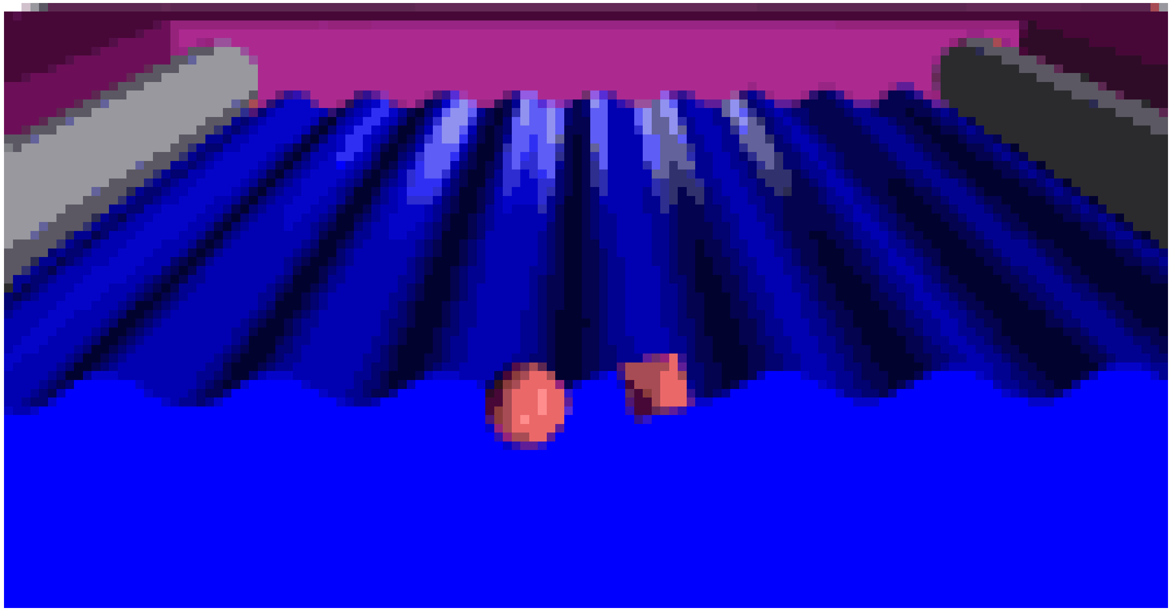


- Note that the total length of the line representing the wave peak
  stays the same where it is being reflected.
  The red part of the incident wave plus the blue part of the reflected wave
  is the same as the original line.
- After reflection, a wave has the same speed,frequency and wavelength. It is only the direction of the wave that has changed



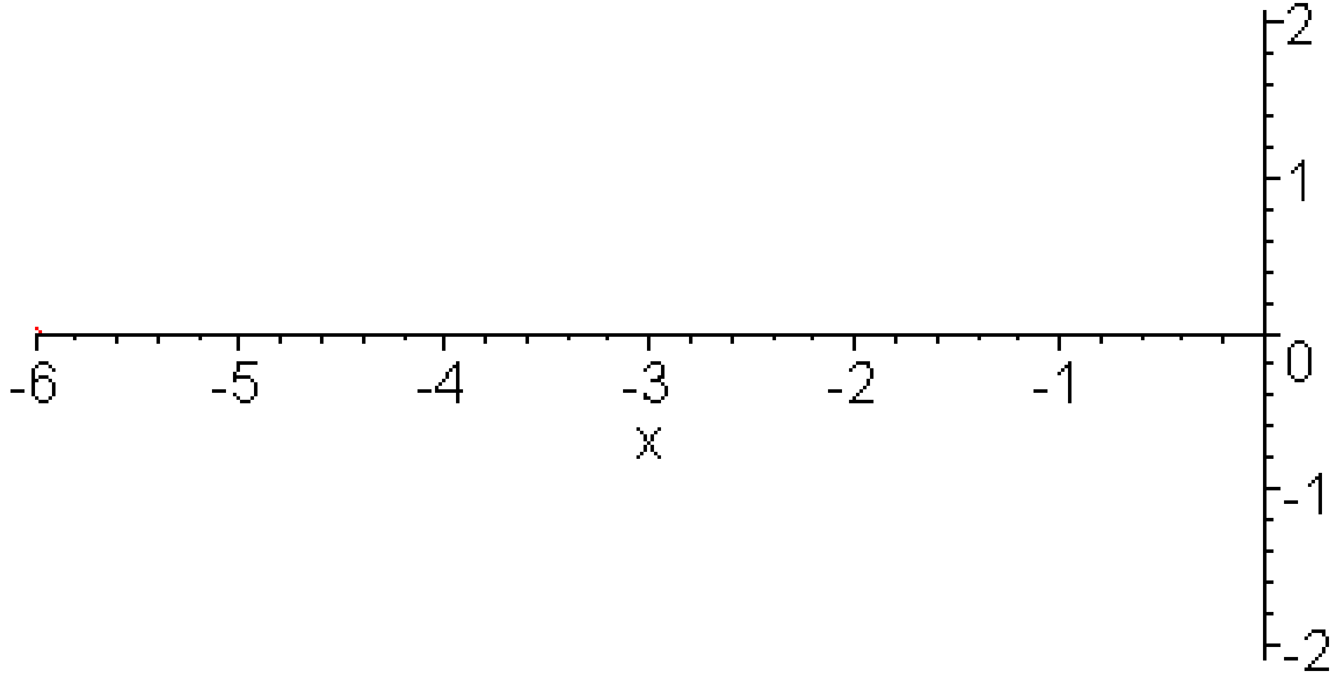
- REFLECTION

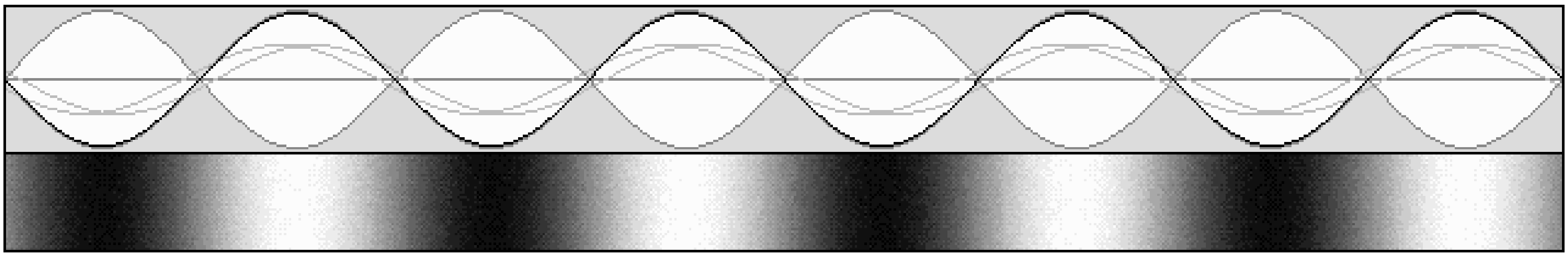
When a beach is steep, the wave fronts get bent and then reflected back. Sometimes part of the energy is absorbed and the remaining energy reflected.



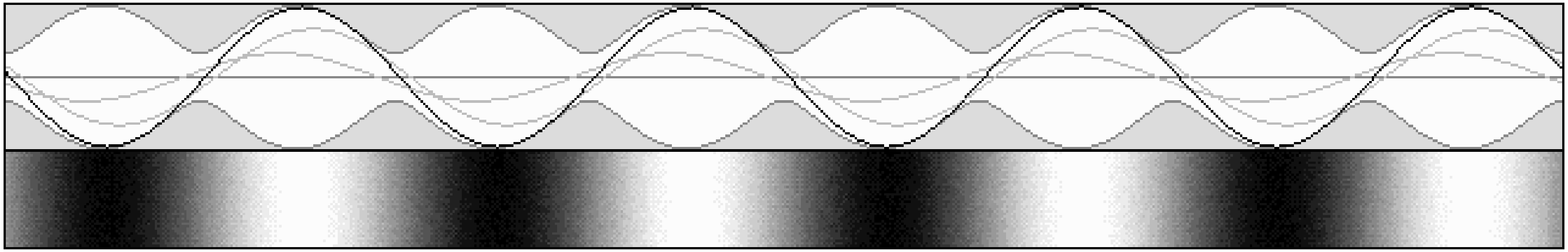
Animation shows the interference of two linear waves propagating in the opposite directions. We can see that small red ball oscillates with the maximal amplitude, while the small red parallelepiped does not oscillate, because it is situated in the node of the standing wave. Parallelepiped is just rotating

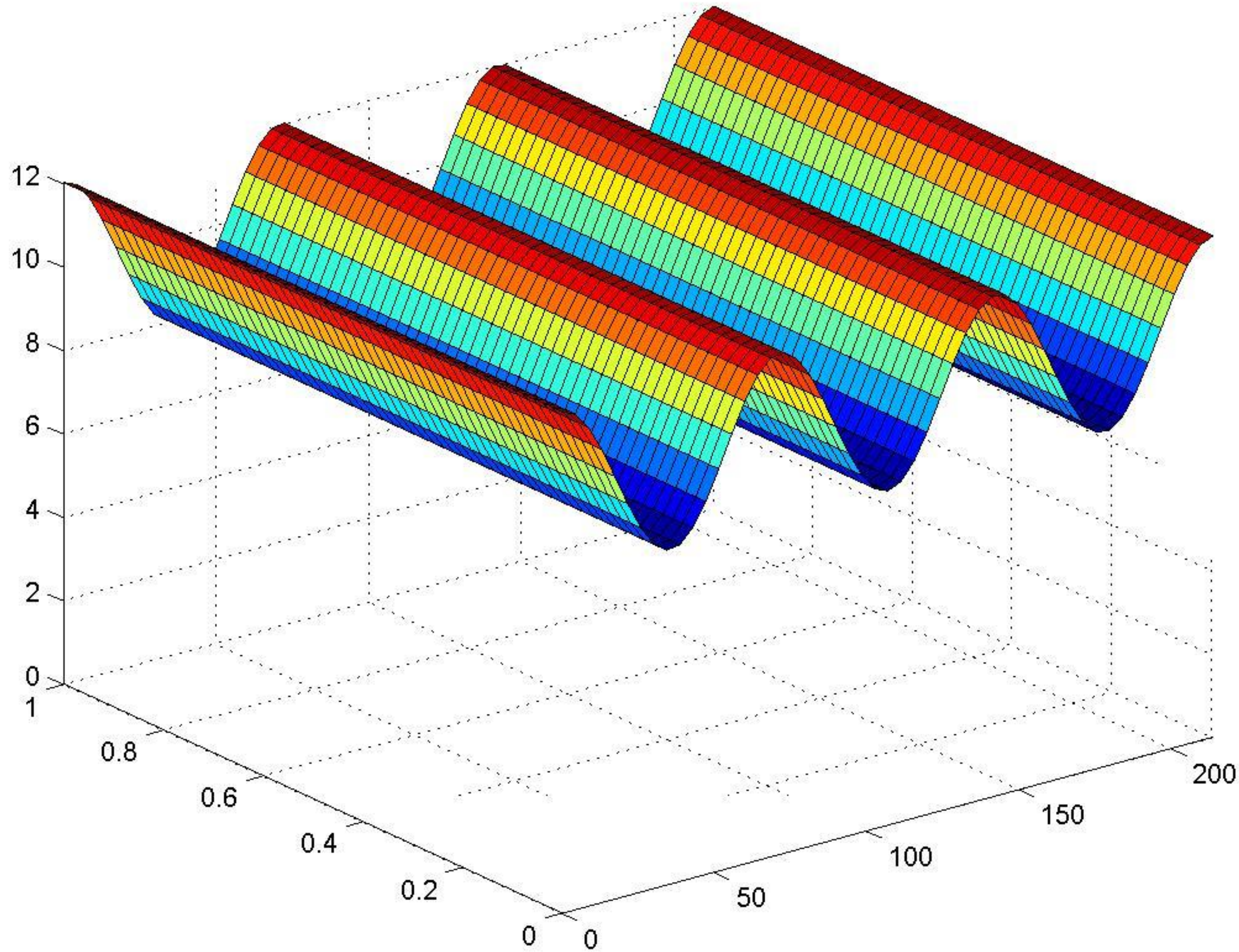
STANDING WAVES – FULL REFLECTION





PARTIALLY STANDING WAVE- PARTIAL REFLECTION





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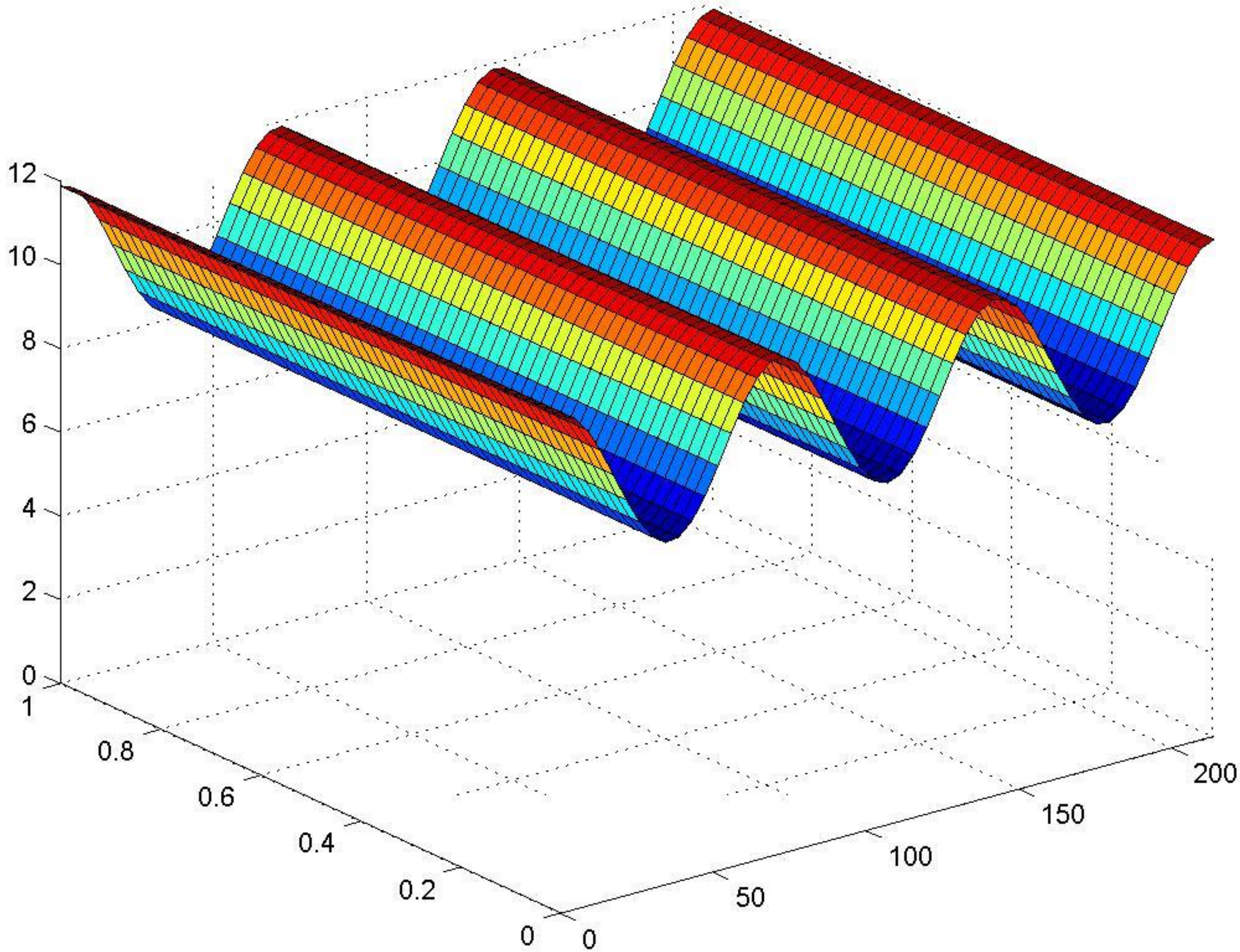

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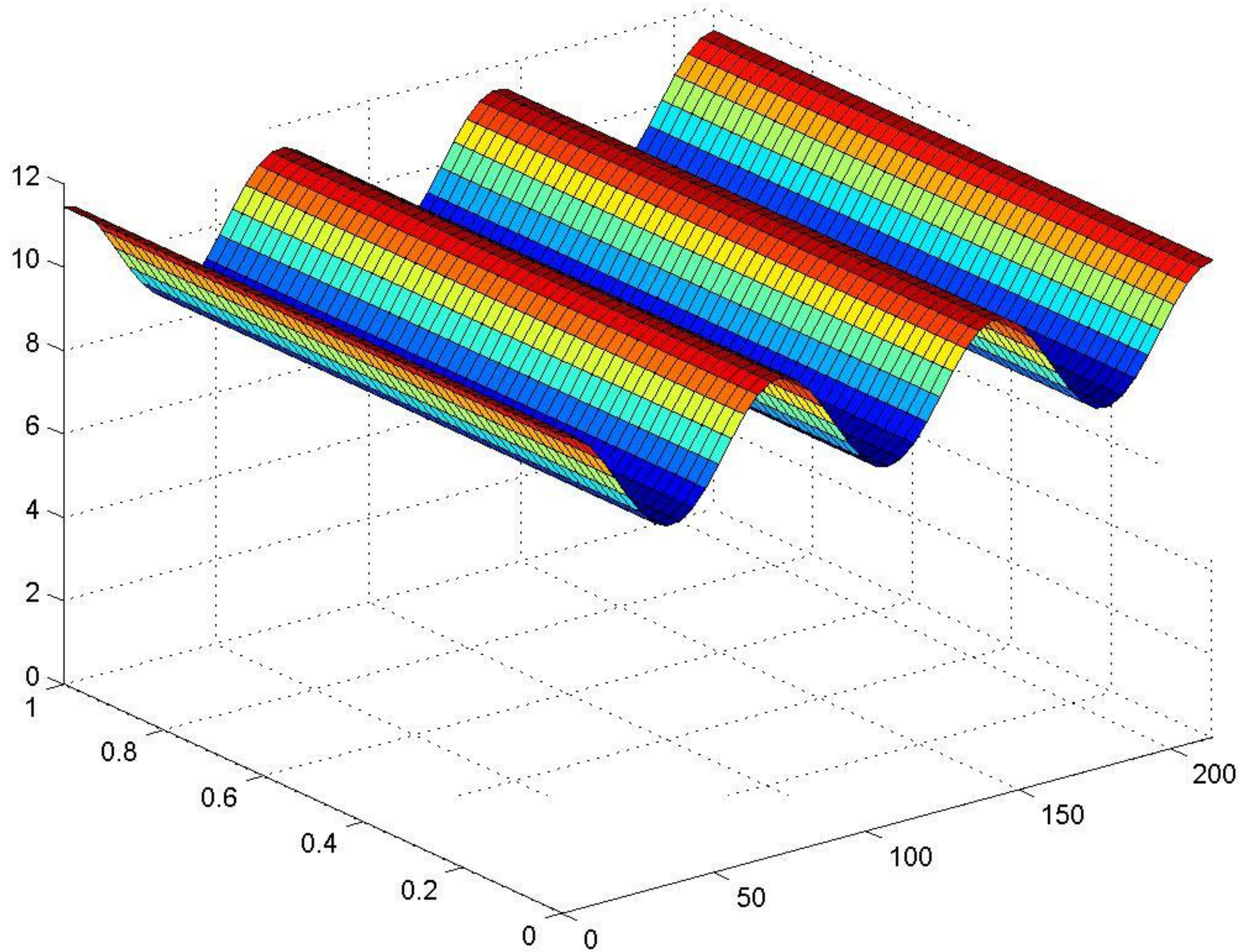

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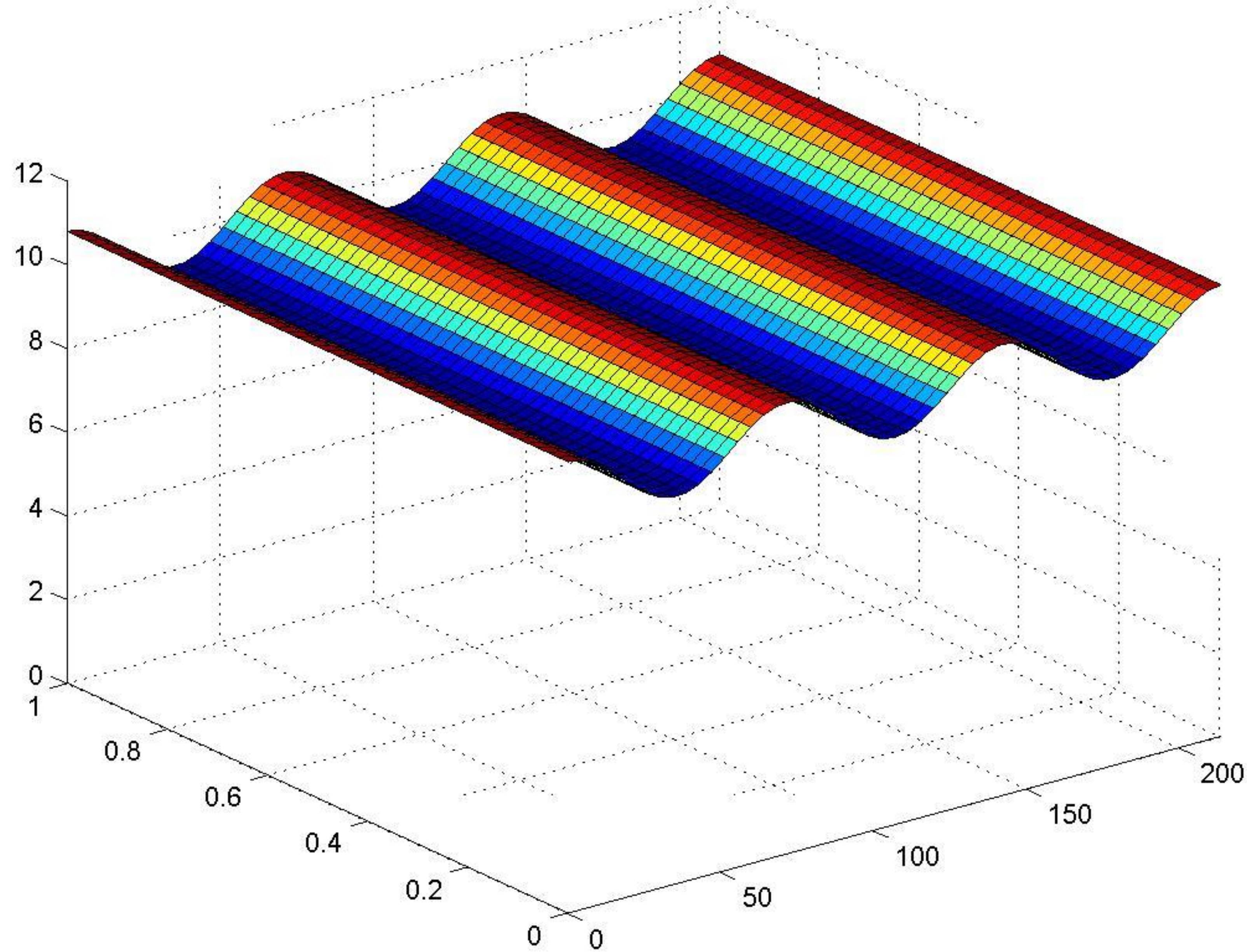

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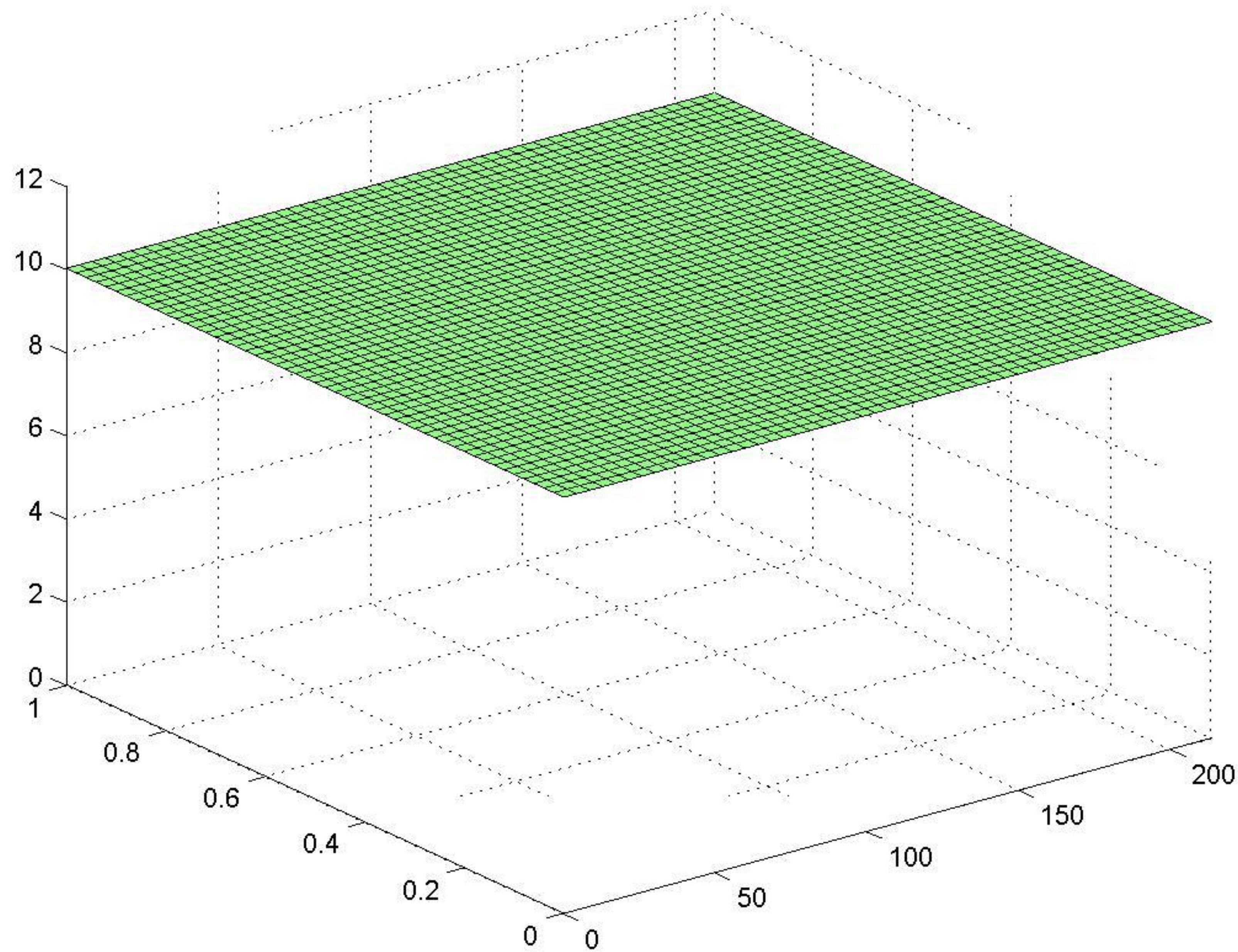

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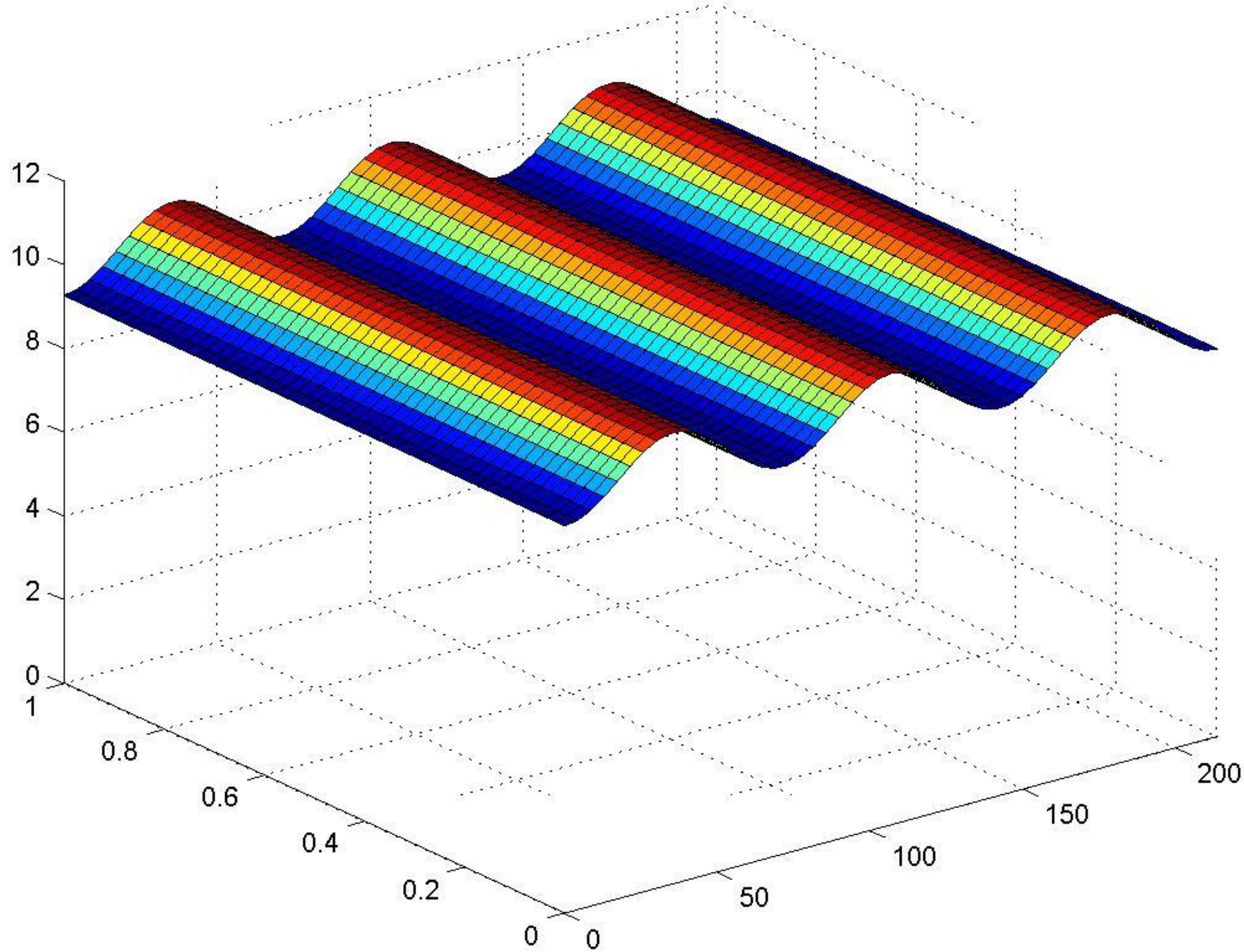

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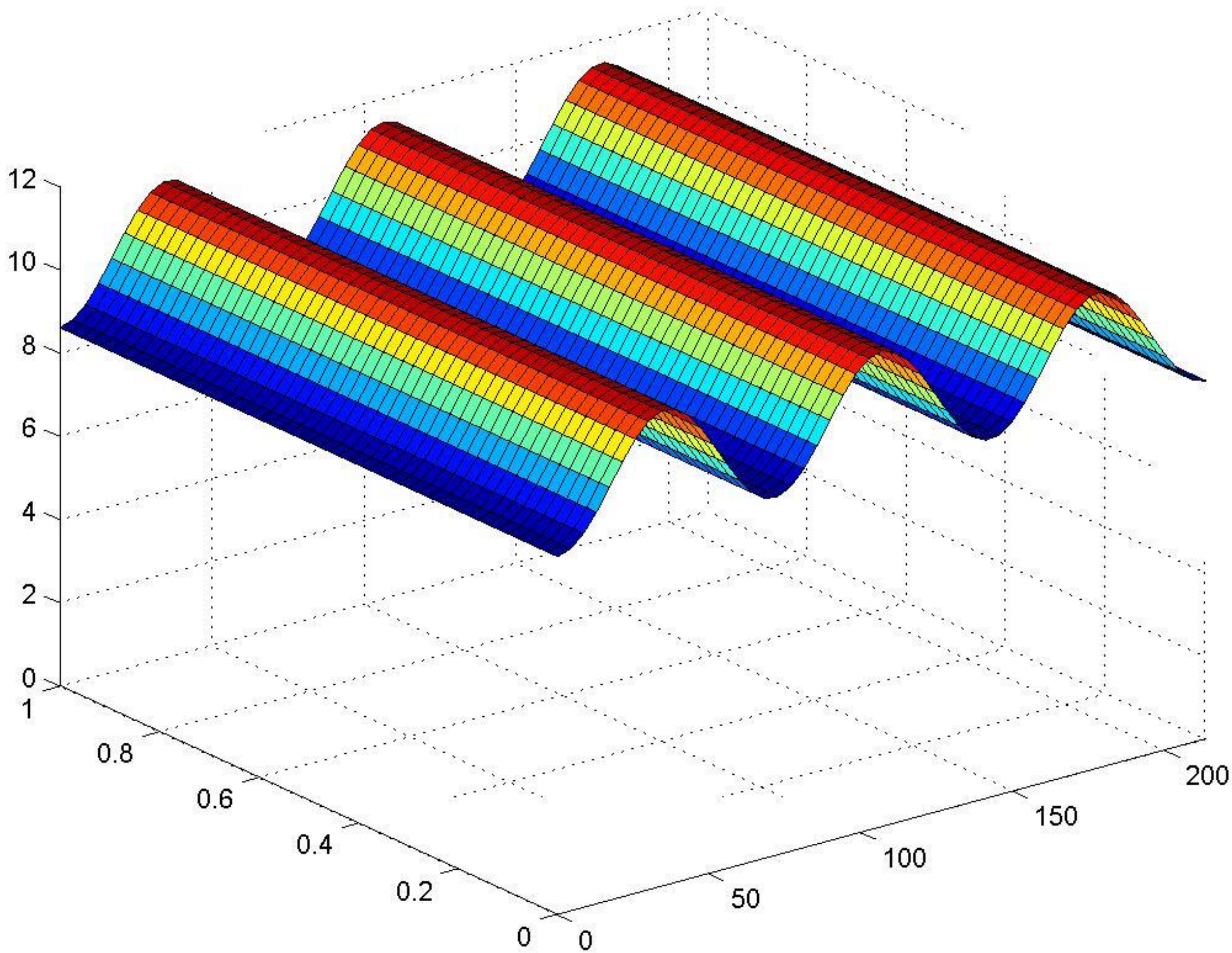

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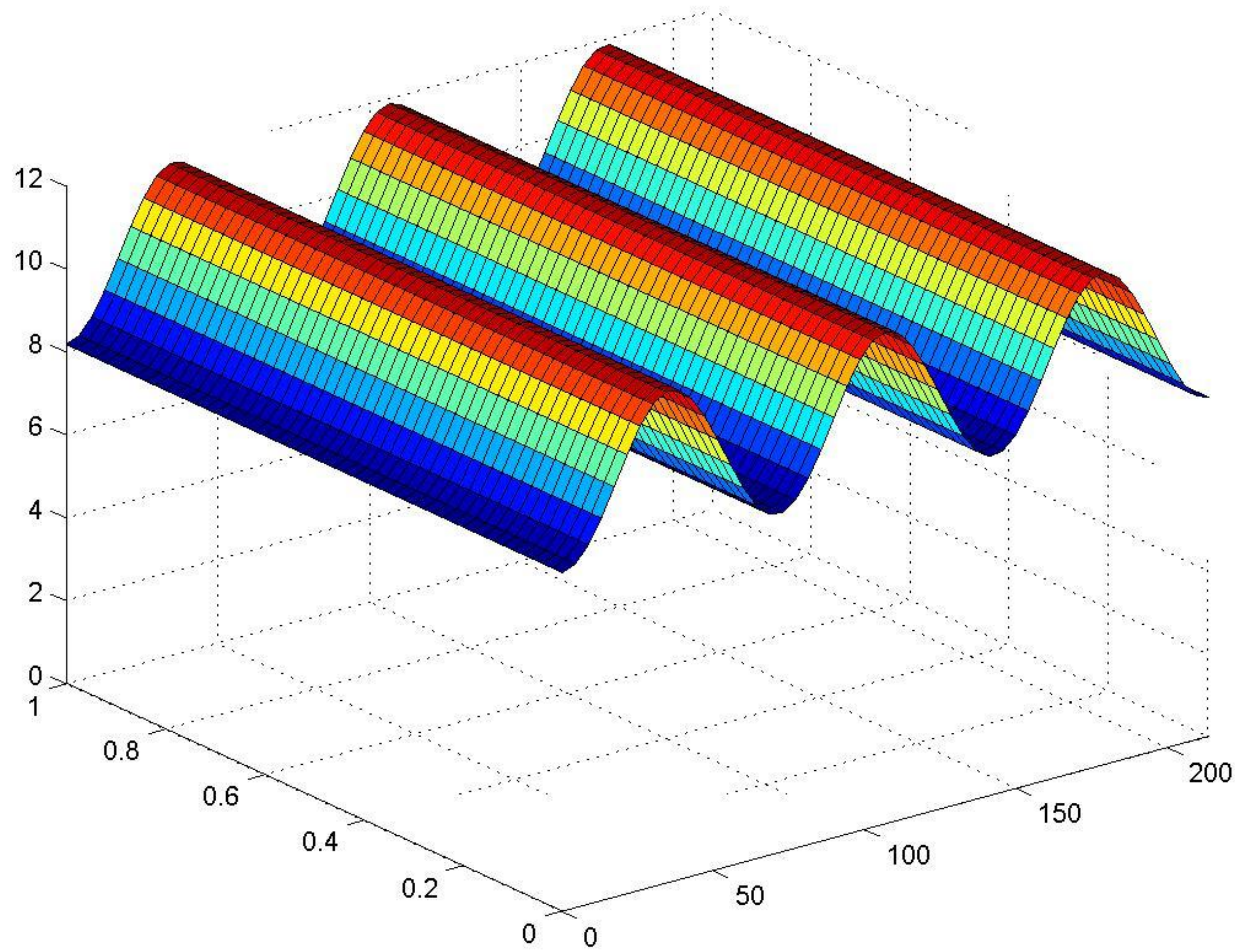

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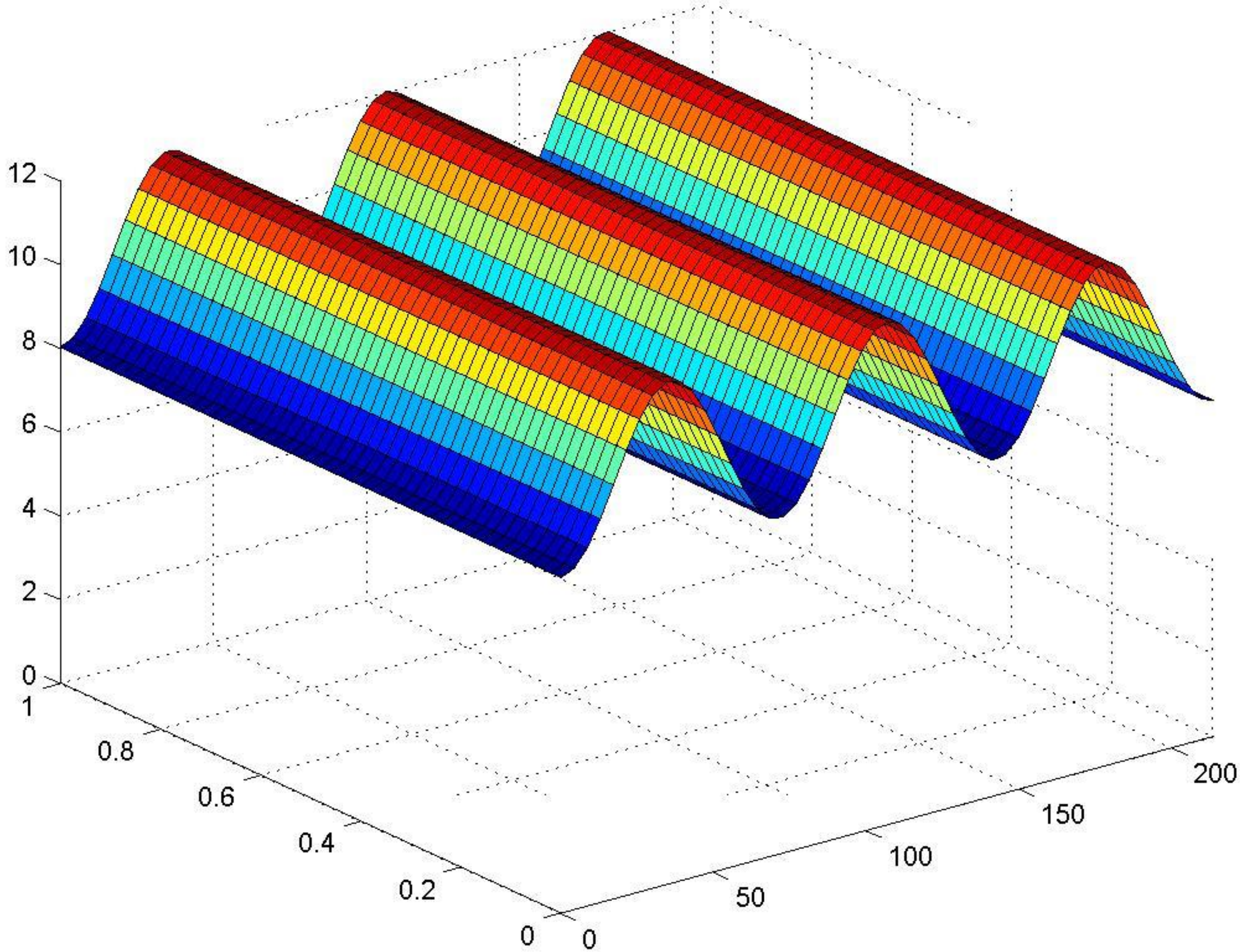
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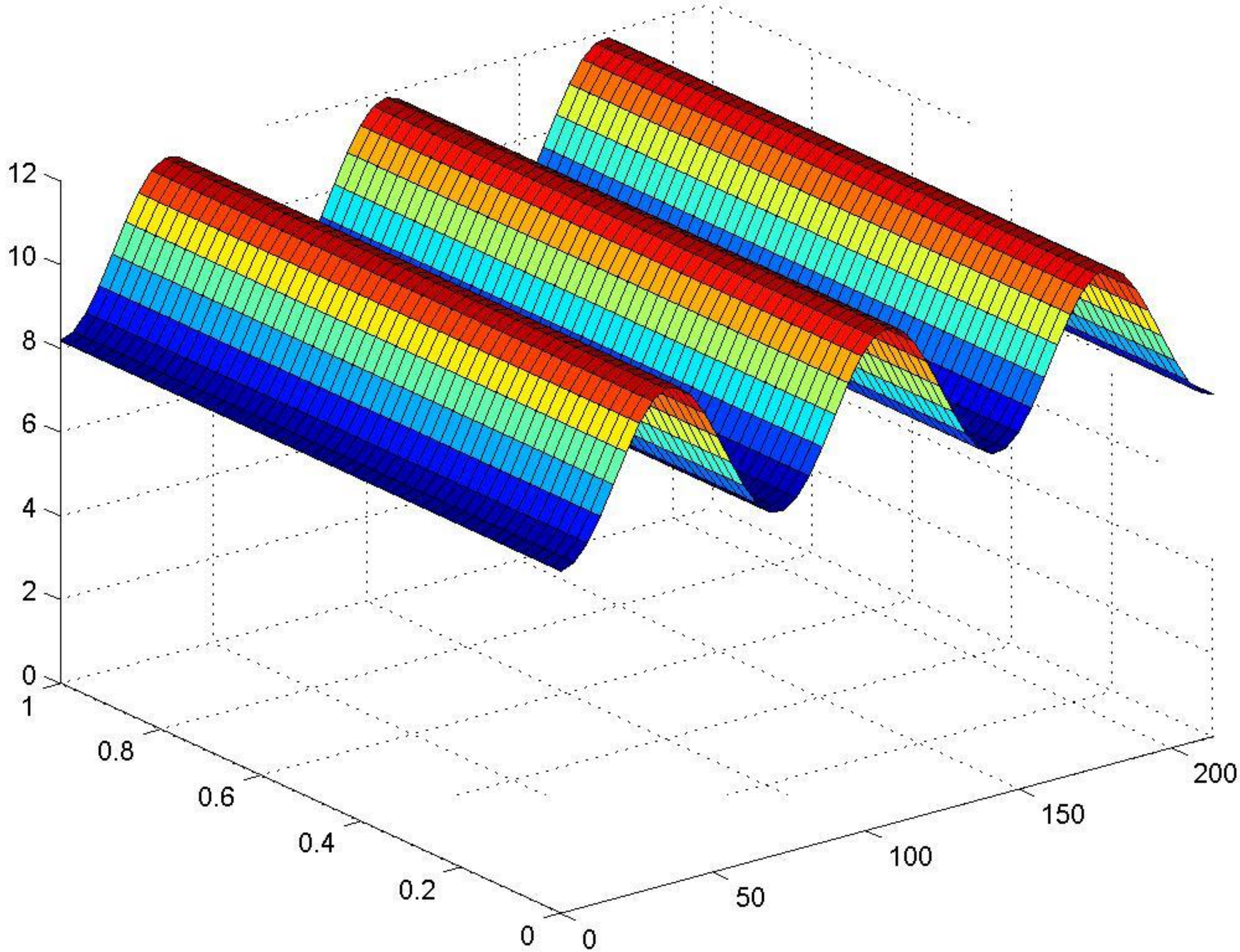

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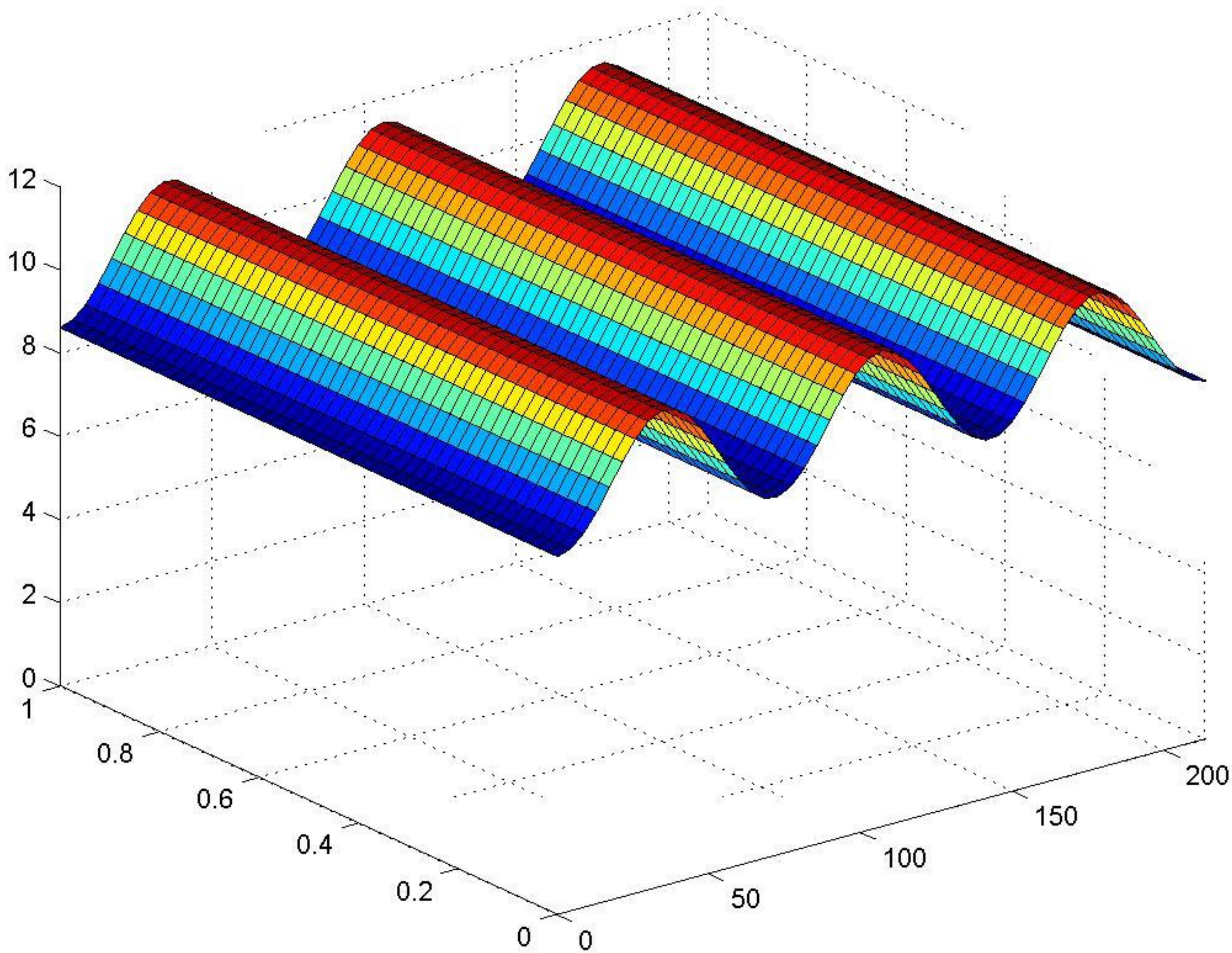
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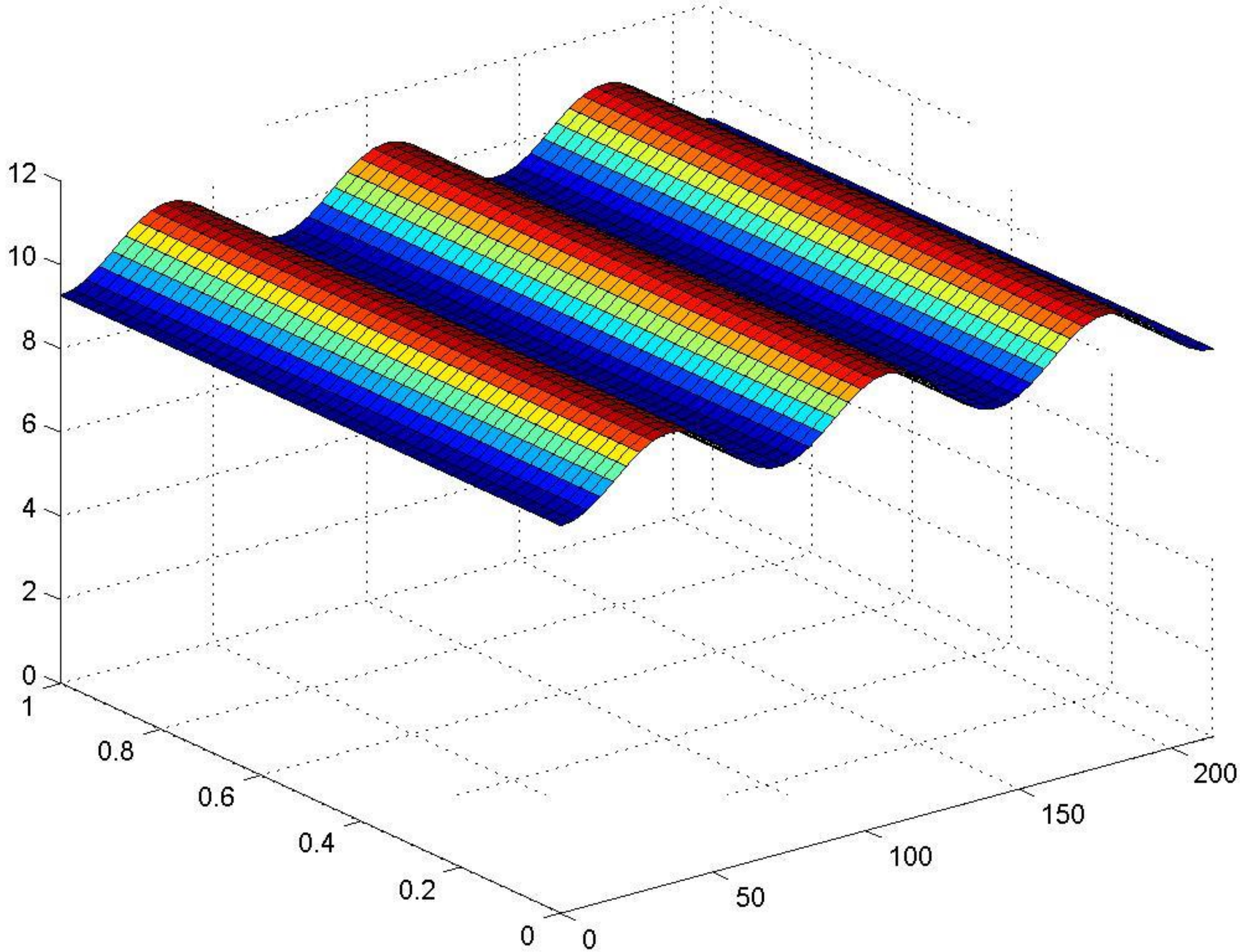
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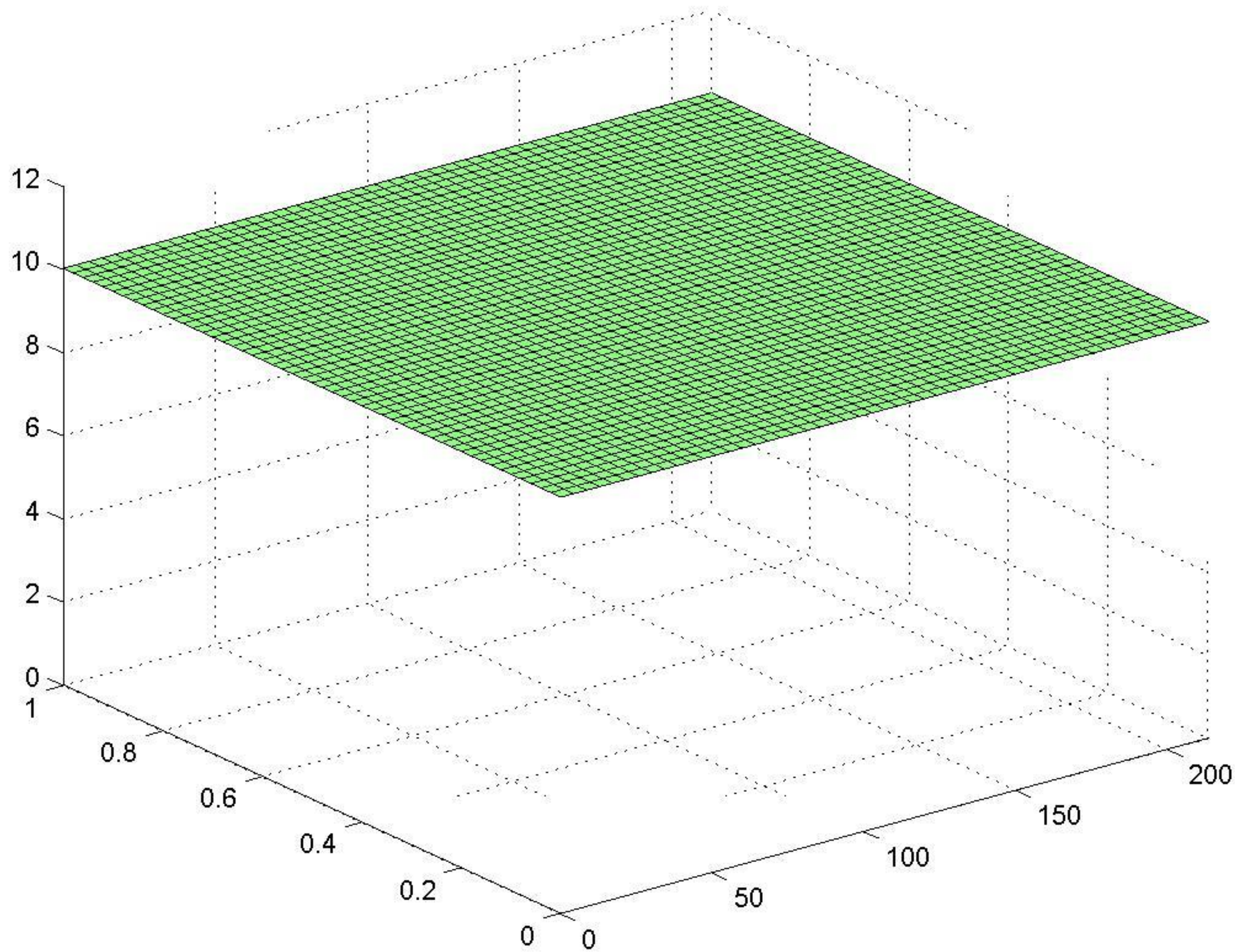

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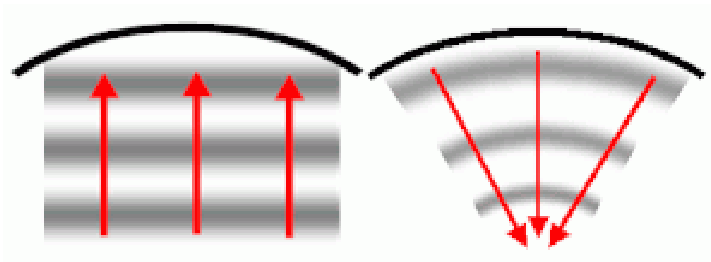
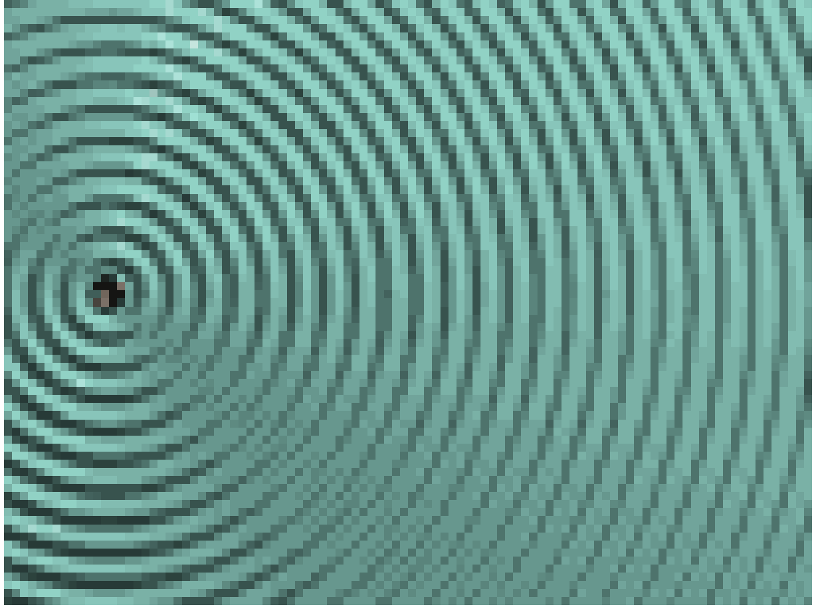
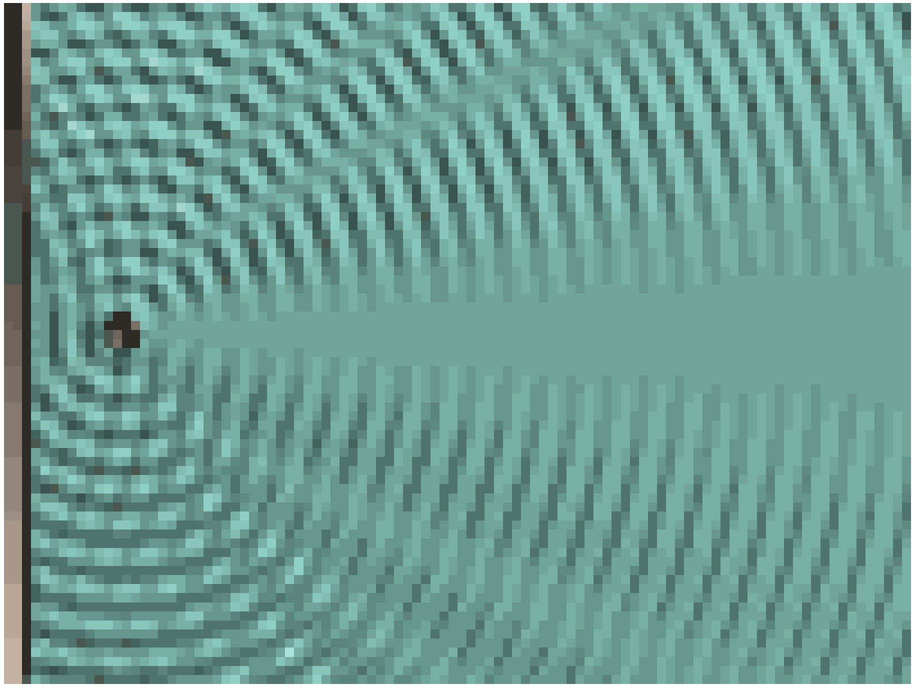






STANDING WAVES – FULL REFLECTION

Wave motion in front of a perfectly reflecting vertical wall subjected to monochromatic waves moving in a direction perpendicular to the barrier can be determined by superposing two waves with identical wave numbers, periods and amplitudes but traveling in opposite directions. The water surface of the incident wave is given to a first-order (linear) approximation by equation

$$\eta_i = \frac{H_i}{2} \cos\left(\frac{2\pi x}{L} - \frac{2\pi t}{T}\right)$$ Incident(incoming) waves

and the reflected wave by

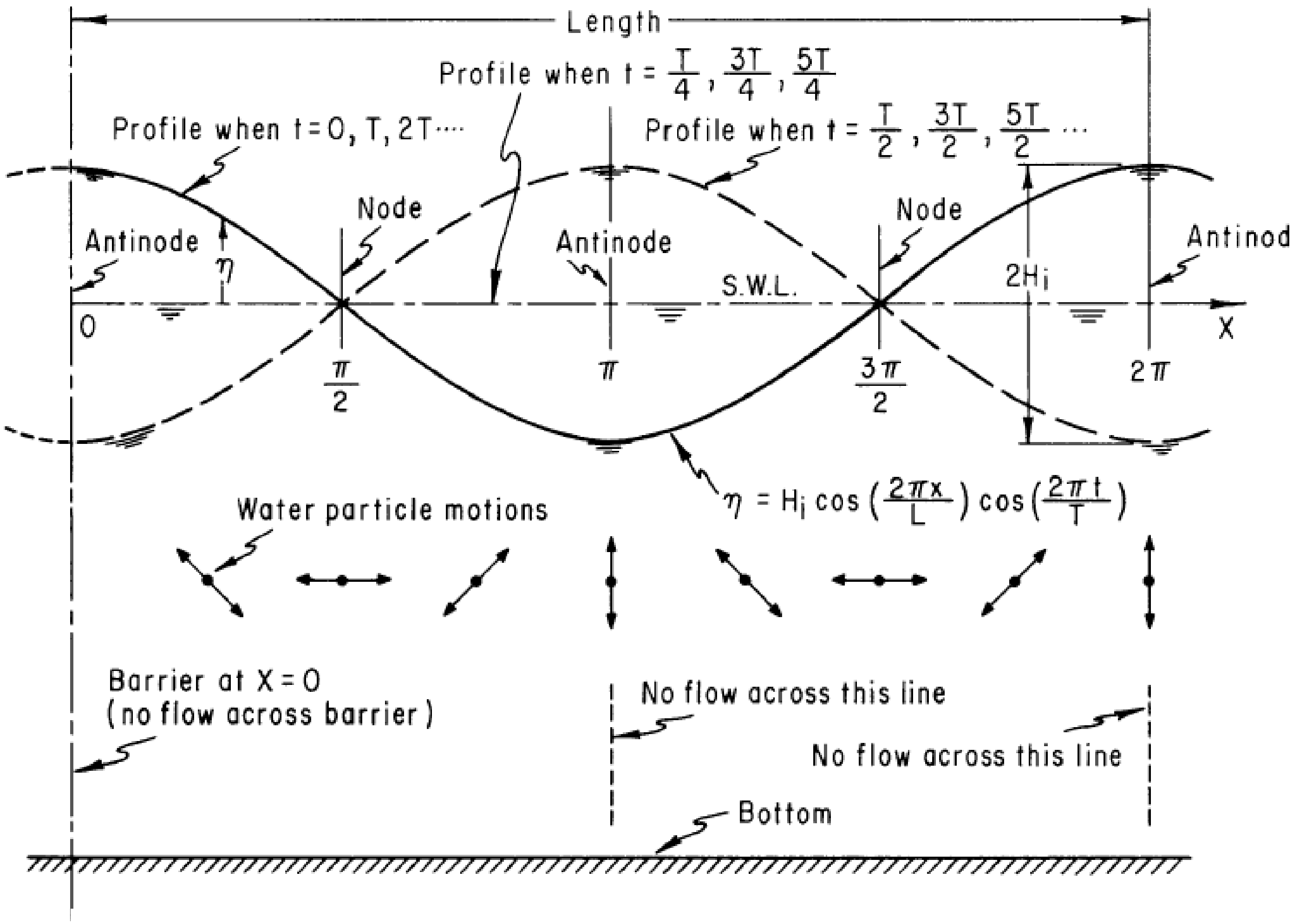
$$\eta_r = \frac{H_r}{2} \cos\left(\frac{2\pi x}{L} + \frac{2\pi t}{T}\right)$$ Reflected waves

Consequently, the water surface is given by the sum of η_i and η_r, or, since $\boxed{H_i = H_r}$,

$$\eta = \eta_i + \eta_r = \frac{H_i}{2}\left[\cos\left(\frac{2\pi x}{L} - \frac{2\pi t}{T}\right) + \cos\left(\frac{2\pi x}{L} + \frac{2\pi t}{T}\right)\right]$$

which reduces to

$$\boxed{\eta = H_i \cos\frac{2\pi x}{L} \cos\frac{2\pi t}{T}}$$



Standing wave (clapotis) system, perfect reflection from a vertical barrier, linear theory.

Reflection coefficient, Cr is the ratio between the reflected wave height, Hr, and incoming wave height Hi

$$\boxed{\mathbf{Cr = \frac{Hr}{Hi}}}$$

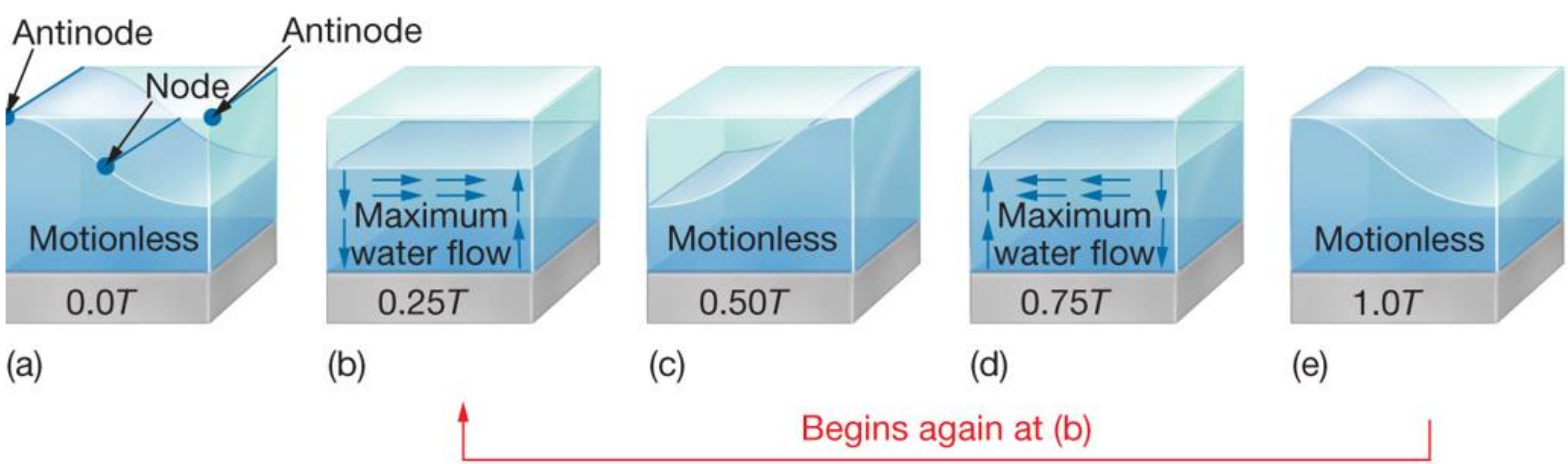
The value of Cr is in between 0 and 1

$$\mathbf{0 \leq Cr \leq 1}$$

Full(complete) reflection	Cr=1
Partial reftection	Cr **≤1**

Standing Waves

- Two waves with same wavelength moving in opposite directions
- Water particles move vertically and horizontally
- Water sloshes back and forth



WAVE REFLECTION

The ratio between the reflected H_r and incident wave heights, H_i, is given by:

$$C_r = \frac{H_r}{H_i} \qquad 0 \le C_r \le 1$$

where C_r is the reflection coefficient.

a) Standing waves: $\eta = \eta_i + \eta_r$, $H_i = H_r = H$, $C_r = 1.0$, $\eta_r = H\cos(kx)\sin(\sigma t)$

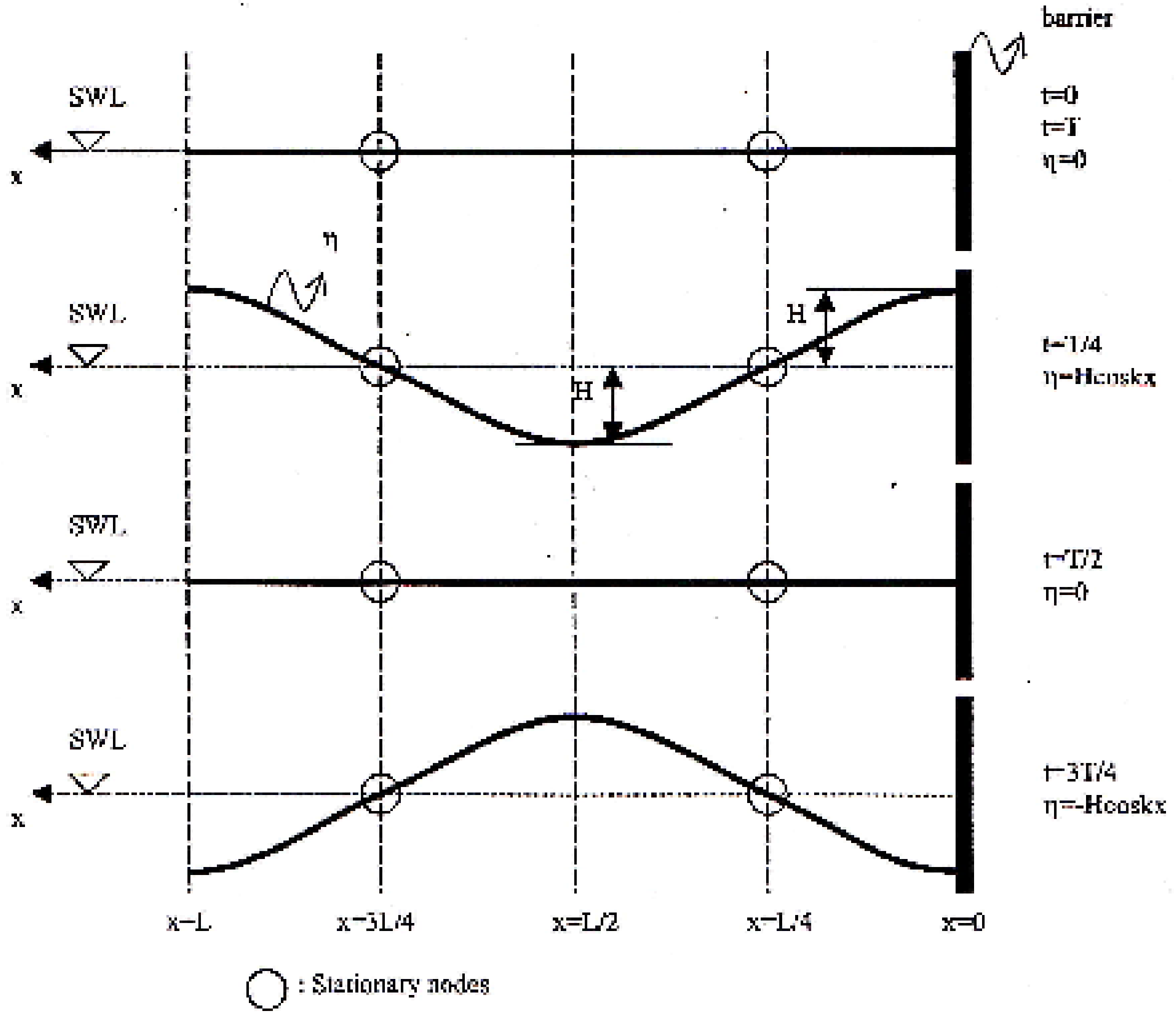


Figure: Resulting profile of 100% wave reflection from a barrier (loss of energy=0)

Barrier is vertical, frictionless, rigid; incident wave directionis perpendicular to barrier;

STANDING WAVE - FULL REFLECTION

Reflection Coefficient: $C_r = H_r / H_i$

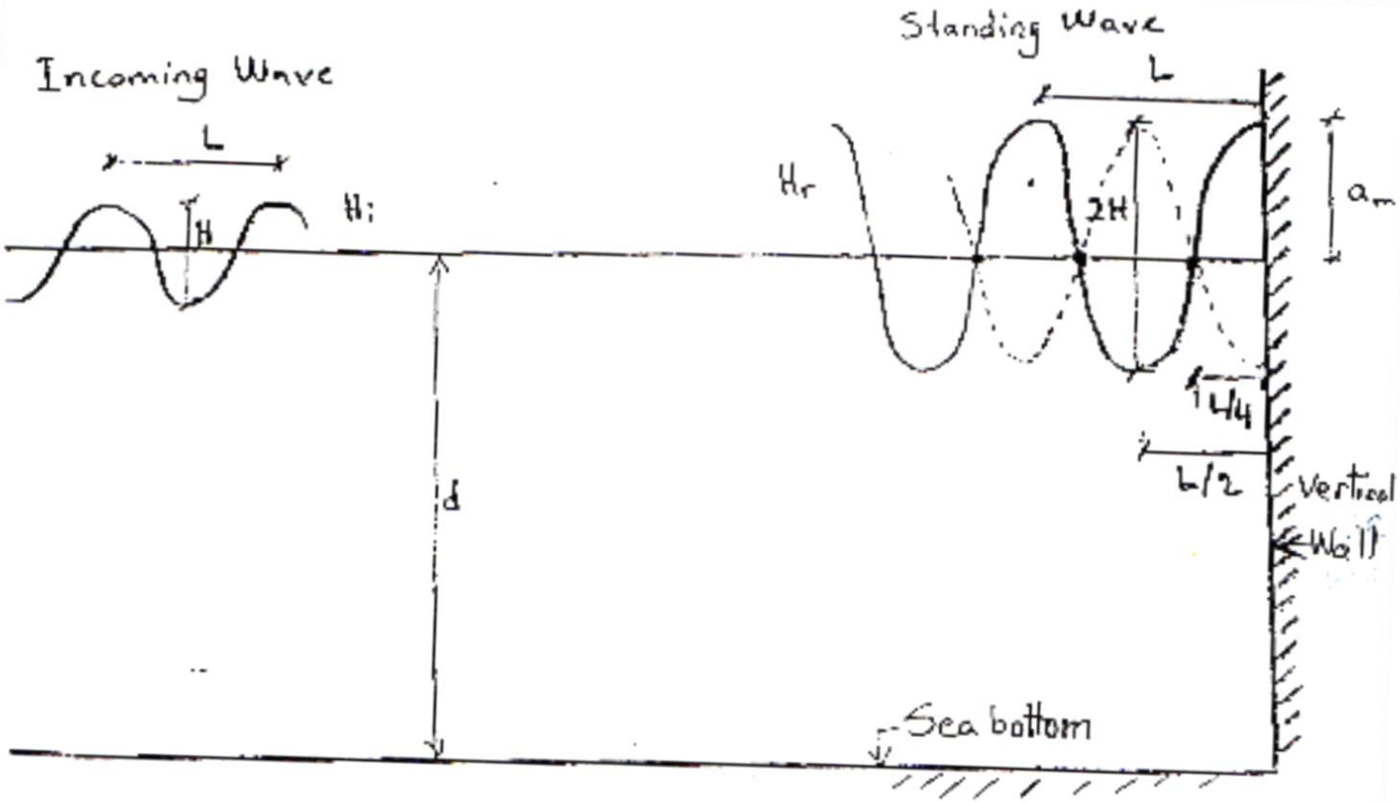
H_i = Incoming wave height
H_r = Reflected wave height

Superposed wave: Standing wave

$$\eta = 2a\cos kx \sin\sigma t = H\cos kx \sin\sigma t$$
$$C_r = 1.0 \dashrightarrow H_i = H_r = H$$

At: On the wall, L/2, L away from the wall $\eta = \pm a_{max}$ (antinodes)

At: L/4, 3L/4 $\eta = 0$ (nodes)



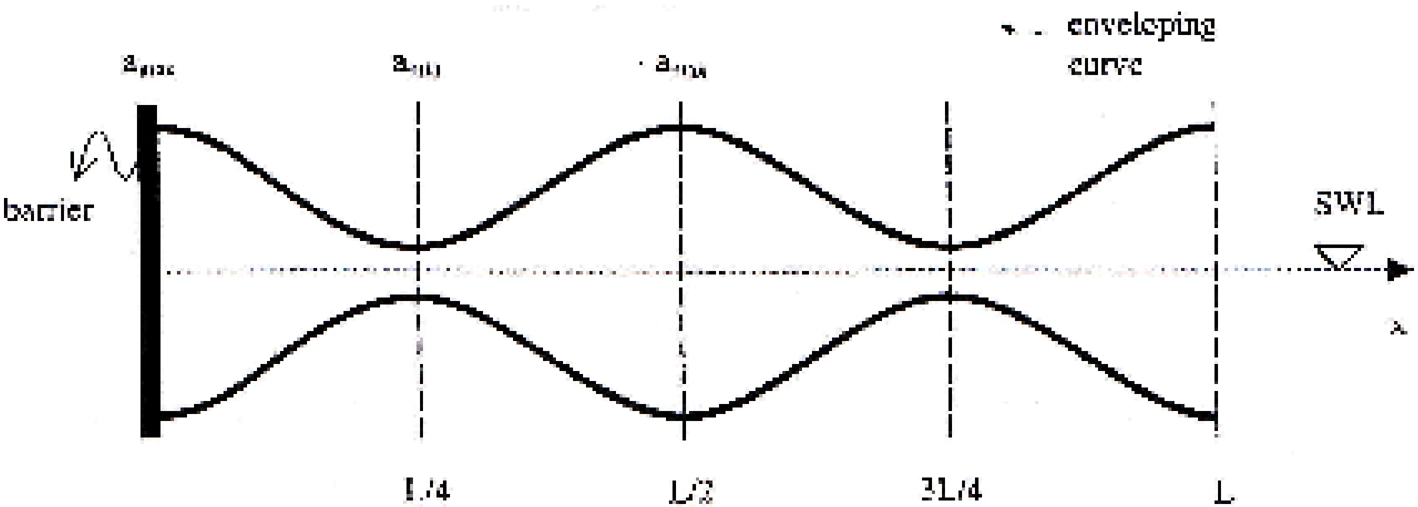
BARRIER: VERTICAL, RIGID, IMPERVIOUS
NO ENERGY LOSS

PARTIALLY REFLECTED WAVE

$$\eta = \left(\frac{H_i}{2} + \frac{Hr}{2}\right)\cos kx \sin \sigma t - \left(\frac{H_i}{2} - \frac{Hr}{2}\right)\sin kx \cos \sigma t$$

$$a_{max} = \left(\frac{H_i}{2} + \frac{Hr}{2}\right) = a_i + a_r$$

$$a_{min} = \left(\frac{H_i}{2} - \frac{Hr}{2}\right) = a_i - a_r$$



EXAMPLES ON REFLECTION

1. Waves of H_0=4 m. with T=7 sec. are approaching towards a vertical rigid sea wall constructed at a depth of 8 m. where reflection coefficient is C_r=1 (on the wall). Take refraction coefficient as K_r=1.0

a- Define the superposed wave.

b- What is the energy loss during reflection?

c- What is the wave length of the standing wave formed?

d- Compute a_{max} at the wall, at L/2 and L distance away from the wall.

a) T=7 sec → L_0=76.44 m.

d/L_0=8/76.44=0.1 (ID) ------>d/L=0.141 →L=56.74 m.

K_s=0.933 →H_s=4*0.933*1.0=3.73 m.

$k=2\pi/L=0.111$

$\sigma=2\pi/T=0.898$

$\eta=2a\cos kx \sin\sigma t$= 3.73Cos(0.111x)Sin(0.898t) ; standing wave

b) C_r=1.0 → Full reflection → No energy loss.

c) L=56.74 m.

d) a_{max} at the wall: 3.73 m.

at L/2: 3.73 m.

at L/4: 0 (nodal point)

2. Waves of H_0=4 m. with T=8 sec. are approaching towards a vertical rigid sea wall constructed at a depth of 10 m. where reflection coefficient is C_r=0.9 (on the wall). Consider only shoaling ; K_r (refraction coef. = 1)

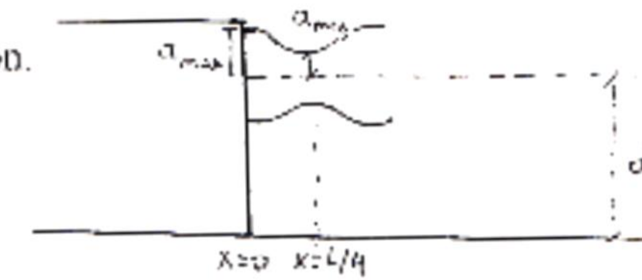
a- Define the superposed wave.

b- Find a_{max} at the wall, at L/2, L distances from the wall.

c- Find a_{min} at L/4, ¾L distances from the wall.

d- Find percent loss of energy during partial reflection.

e- How would you increase the loss of energy?



Solution:

a) Partially reflected wave

b) 1) L_0=100 m. → d/L_0=0.1 GWT → K_s=0.933, L=70.9 m

2) $H=H_0K_sK_r$ =4x1x0.933=3.73 m.

3) $C_r=H_r/H_i$=0.9 → H_r=3.36 m.

4) a_{max} occurs at the wall face, at L/2, L

$a_{max}=a_i+a_r=H_i/2+H_r/2$=3.73/2+3.36/2=3.55 m

c) a_{min} occurs at L/4, ¾L distances from the wall

$a_{min}=a_i-a_r$=0.19 m. at 70.9/4=17.7 m.

d) $H_i/H_r=0.9$ $\frac{E_i-E_r}{E_i}=\frac{H_i^2-H_r^2}{H_i^2} \rightarrow 1-\frac{E_r}{E_i}=1-\left(\frac{H_r}{H_i}\right)^2 \Rightarrow \frac{E_r}{E_i}=(0.9)^2 \Rightarrow 0.81$

$E_r=0.81E_i$. % loss of energy=E_i-E_r=0.19 (19%)

e) By using energy dissipators.

3. Aprogressive wave of H=2 m., T=8 sec is reflected from a vertical, rigid, impervious barrier

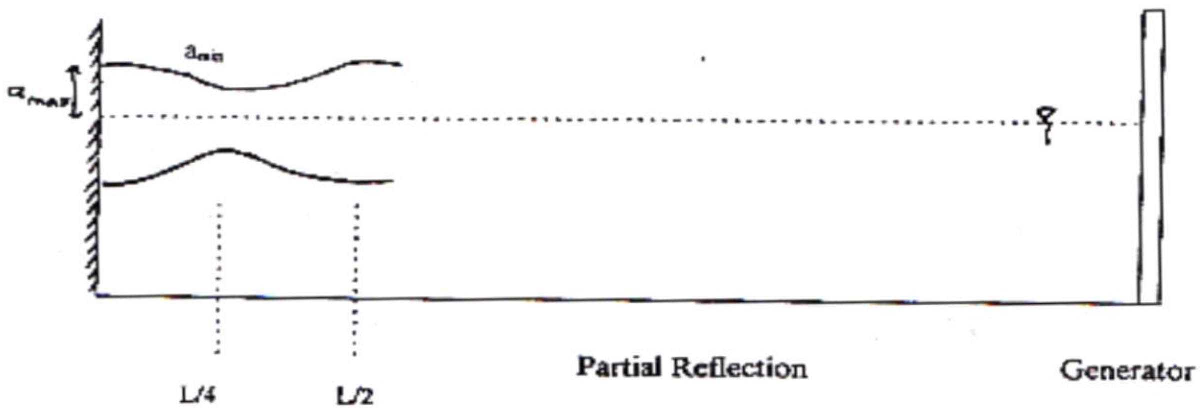
a) Find a_{max} on the barrier if C_r=1.0
b) Find a_{max} on the barrier if C_r=0.8

The wall is constructed at a depth of d=10 m.

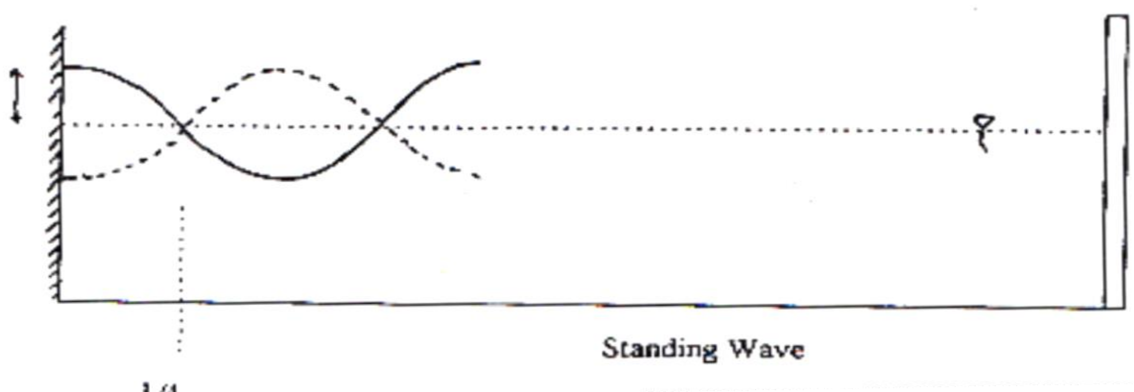
Solution:

a) C_r=1.0 → a_{max} at the barrier = H =2 m.
b) C_r=0.8

$a_{max}=a_i+a_r=1+0.8=1.6$ m. at the wall, L/2, ...
$a_{min}= a_i-a_r=1-0.8=0.2$ m. at L/4, ¾L, ...



Partial Reflection



Standing Wave

HOMEWORK

1. Figure shows the top view and the side view of a wave flume. At the end of the flume, there is a vertical wall where the width of the flume is 1 meter and the water depth is 2 meters. When the wave maker started it generates waves of T = 1 second. As these waves move towards the end of the flume, at section 1 the incoming wave height is measured as H = 10 cm. At section 1, the flume width is 2 meters and the water depth is 1 meter. When the waves reach the end of the flume, they are reflected back from the vertical wall. (Reflection coefficient of the wall is Cr = 0.8)

Compute maximum surface elevation (n) at the face of the wall and at the location L/4 meters away from the vertical wall after the reflection from the wall.