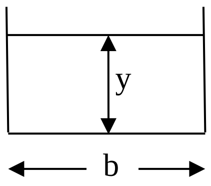
EXERCISE PROBLEMS ON OPEN CHANNEL FLOWS

1) A channel flow cross section has an area of 18 m^2. Calculate its best dimensions if (a) rectangular, (b) trapezoidal with 1 (vert.) on 2 (horiz.) side slopes, and (c) V-shaped.

(a)

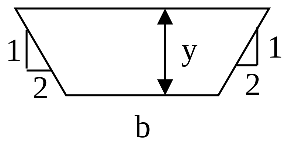


$$A = by \qquad P = b + 2y$$

$$b = \frac{A}{y} \qquad P = \frac{A}{y} + 2y$$

$$\frac{dP}{dy} = 2 - \frac{A}{y^2} = 0 \Longrightarrow 2y^2 = by = 18m^2 \Longrightarrow 2y = b \Longrightarrow y = 3\,\text{m} \quad b = 6\,\text{m}$$

(b)



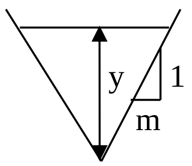
$$A = by + 2y^2$$

$$P = b + 2y\sqrt{5}$$

$$b = \frac{A}{y} - 2y \qquad P = \frac{A}{y} + y\left(2\sqrt{5} - 2\right) \qquad \frac{dP}{dy} = -\frac{A}{y^2} + \left(2\sqrt{5} - 2\right) = 0$$

$$\frac{18}{y^2} = \left(2\sqrt{5} - 2\right) \Longrightarrow y = 2.7\text{ m} \Longrightarrow b = 1.274\text{ m}$$

(c)

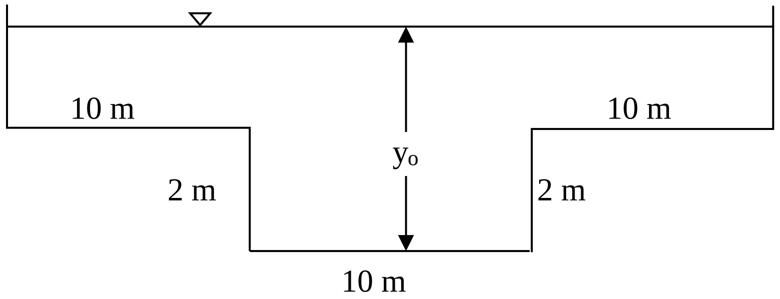


$$\Longrightarrow A = my^2 \qquad y = \sqrt{\frac{A}{m}}$$

$$P = 2y\sqrt{(1+m^2)} \Longrightarrow P = \frac{2\sqrt{A}\sqrt{(1+m^2)}}{\sqrt{m}}$$

$$\frac{dP}{dm} = \sqrt{A}\,\frac{\left(1 - \frac{1}{m^2}\right)}{\sqrt{\left(\frac{1}{m} + m\right)}} \Longrightarrow m = \mp 1 \Longrightarrow y = \sqrt{18} = 3\sqrt{2}m$$

2) This flood channel has a Manning n of 0.017 and a slope of 0.0009. Estimate the depth of uniform flow for a flowrate of 60 m^3/s.



Solution:

$$Q = \frac{A}{n} R^{2/3} \sqrt{S_0} = \frac{1}{n} \frac{A^{5/3}}{P^{2/3}} \sqrt{S_0}$$

$$A = 20 * (y_0 - 2) + 10 * y_0$$

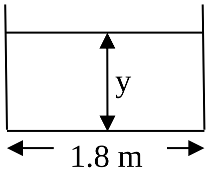
$$P = 2 * y_0 + 30$$

$$\mathrm{Q} = \frac{1}{0.017} \frac{[20 * (y_0 - 2) + 10 * y_0]^{5/3}}{[2 * y_0 + 30]^{2/3}} \sqrt{0.0009}$$

$$y_0 = 2.48m$$

3) At what depths may 0.85 m^3/s flow in a rectangular channel 1.8 m wide if the specific energy is 1.2 m?

Solution:

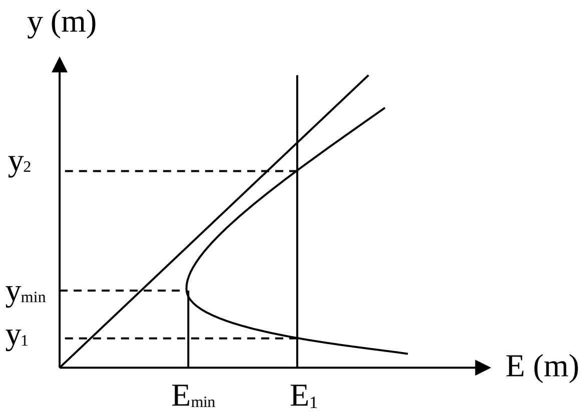


$\Longrightarrow$ $Q = 0.85 m^3/s$ $\quad$ $q = \frac{Q}{b} = \frac{0.85}{1.8} = 0.4722\, m^3/s$

Critical water depth $\Longrightarrow$ $y_c = \sqrt[3]{\frac{q^2}{g}} = \sqrt[3]{\frac{0.4722^2}{9.81}} = 0.2833\, m$

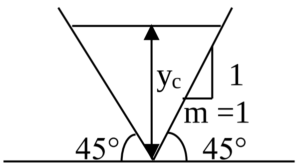
$$E = y + \frac{V^2}{2g} = y + \frac{Q^2}{A^2 \times 19.62} = y + \frac{0.85^2}{(1.8y)^2 \times 19.62} = 1.2m$$

$\Longrightarrow$ $y_1 = 0.102m$ $\quad$ $y_2 = 1.192m$ (two different water depths may occur in the channel depends on the state of flow)



4)Derive an expression for critical depth for an open channel of V-shaped cross section with side slopes of 45°. What is the ratio between critical depth and minimum specific energy for this channel?

Solution:



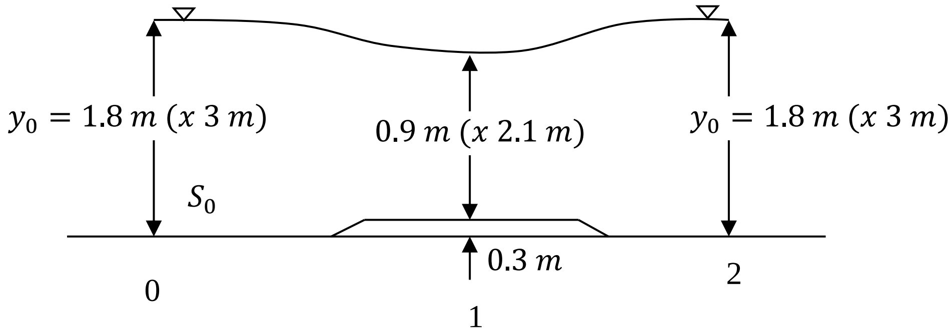
$$A = my^2 = y^2 \quad \Longrightarrow \quad y_c = \sqrt{A_c} \qquad T_c = 2y_c$$

$$\frac{Q^2}{g} = \frac{A_c^3}{T_c} \quad \Longrightarrow \quad Q^2 = \frac{A_c^3}{T_c} \times g$$

$$E_{min} = y_c + \frac{Q^2}{A_c^2 \times 2g} = y_c + \frac{\frac{A_c^3}{T_c} \times g}{A_c^2 \times 2g} = y_c + \frac{A_c}{2T_c}$$

$$A_c = y_c^2 \quad \& \quad T_c = 2y_c \quad \Longrightarrow \quad E_{min} = y_c + \frac{y_c^2}{4y_c} = \frac{5y_c}{4} \quad \Longrightarrow \quad \frac{y_c}{E_{min}} = \frac{4}{5}$$

5) Is this water surface profile possible? If not, what profile is to be expected? The channel and construction are both of rectangular cross section. The flowrate is 7.2 m^3/s.



Solution:

$$Q = 7.2 m^3/s \Longrightarrow q_0 = q_2 = \frac{Q}{b} = \frac{7.2}{3} = 2.40 m^3/s$$

Critical water depth $\Longrightarrow$ $y_{c0} = y_{c2} = \sqrt[3]{\frac{q^2}{g}} = \sqrt[3]{\frac{2.40^2}{9.81}} = 0.837\,m$

$$E_0 = E_2 = y + \frac{V^2}{2g} = y + \frac{Q^2}{A^2 \times 19.62} = 1.8 + \frac{7.2^2}{(1.8 \times 3)^2 \times 19.62} = 1.8906\,m$$

$y_0 = y_2 = 1.8m > y_c = 0.837\,m$ $\Longrightarrow$ The flow is subcritical...

<u>Energy equation between 0&1</u>

$$E_0 = E_1 + \Delta z + \Delta h$$

$$y_0 + \frac{V_0^2}{2g} = y_1 + \frac{V_1^2}{2g} + \Delta z + \Delta h$$

$$1.8906 = 0.9 + \frac{7.2^2}{(0.9 \times 2.1)^2 \times 19.62} + 0.30 + \Delta h$$

$1.8906 \neq 1.9397 + \Delta h$ Even if hydraulic losses are neglected; this water surface profile cannot be possible...

Critical water depth at the section 1 $\Longrightarrow$ $y_{c1} = \sqrt[3]{\frac{q^2}{g}} = \sqrt[3]{\frac{(7.2/2.1)^2}{9.81}} = 1.062\,m$

$$E_{1\min} = y_{c1} + \frac{Q^2}{A_{c1}^2 \times 19.62} = 1.062 + \frac{7.2^2}{(1.062 \times 2.1)^2 \times 19.62} = 1.593\,m$$

$E_1 = E_0 - \Delta z = 1.8906 - 0.3 = 1.5906\,m$ $\Longrightarrow$ $E_1 \geq E_{1\min} \geq 1.593m$ (It must be bigger than minimum energy)

E_1 is to be 1.593 m, so <u>y_1 will be the critical water depth</u> at the section 1 that is **y_1=1.062 m**.

Thus, the energy equation between 0 & 1 (Δh=0 is assumed)

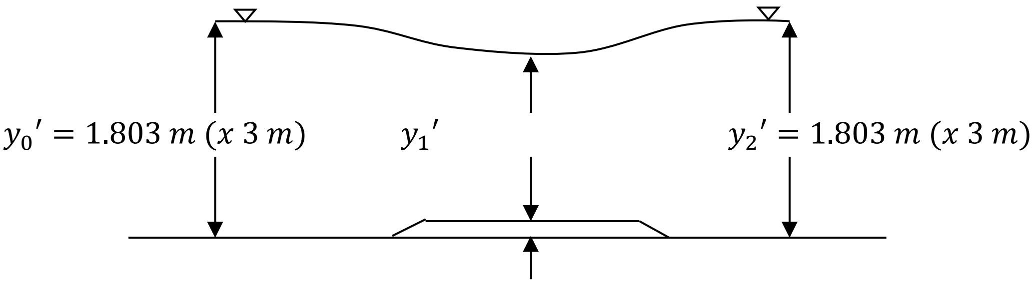
$$E_0' = E_1 + \Delta z + \Delta h$$

$$E_0' = y_1 + \frac{V_1^2}{2g} + \Delta z$$

$$E_0' = 1.062 + \frac{7.2^2}{(1.062 \times 2.1)^2 \times 19.62} + 0.30 = 1.893\text{m}$$

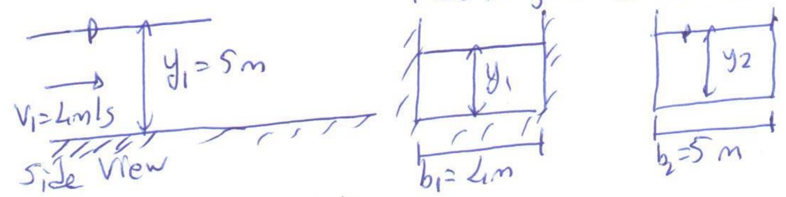
$$E_0' = y_0' + \frac{V_0'^2}{2g} = 1.893\text{m}$$

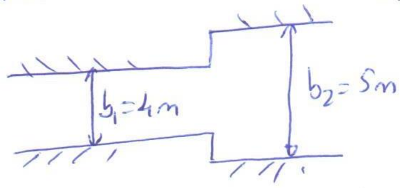
$y_0' = y_2' =$ **1.803 m** (After Choking)



6)

rectangular channel





Top view

Calculate $y_2 = ?$ Draw E vs y

SOLUTION:

$q_1 = V_1 \cdot y_1 = 4 \cdot 5 = 20\ m^2/s$ $\quad Q = 20 \cdot 4 = 80\ m^3/s = q_1 \cdot b_1 \rightarrow q_2 = \frac{Q}{b_2} = \frac{80}{5}$

$\boxed{q_2 = 16\ m^2/s}$

$q_2 = \frac{Q}{b2} = \frac{80}{5} = 16\ m^2/s$

$E_1 = y_1 + \frac{V_1^2}{2g} = 5 + \frac{4^2}{19.62} = 5.82m$

Horizontal bottom $E_1 = E_2$ $\quad (Fr)_1 = \frac{V_1}{\sqrt{g y_1}} = 0.57 < 1$ subcritical flow

$\therefore E_2 = 5{,}82m$

$E_2 = 5{,}82 = \boxed{y_2} + \frac{q_2^2}{2g\, y_2^2}$

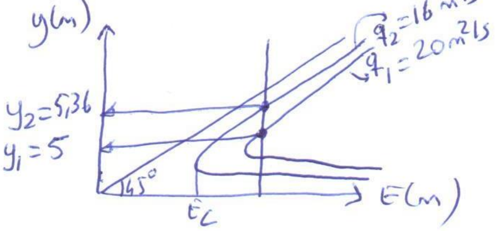
$y_2 = 5.82 - \frac{13.048}{y_2^2}$

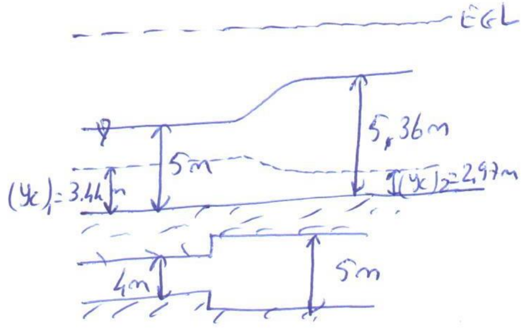
$(y_c)_1 = \sqrt[3]{\frac{q_1^2}{9.81}} = 3.44m$

$(y_c)_2 = \sqrt[3]{\frac{q_2^2}{9.81}} = 2{,}97m$

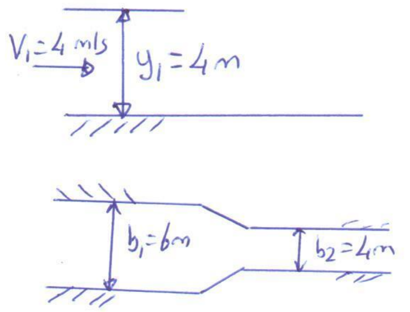
$(y_2)_{ass}$	$(y_2)_{cal}$
5	5,3
5,3	5,35
5,35	5,36
5,36	5,36 ✓

$\rightarrow y_2 = 5{,}36m$





7)



Rectangular channel
Calculate $y_2 = ?$
Draw E vs y and water surface profile

SOLUTION

$q_1 = V_1 \cdot y_1 = 4 \times 4 = 16\ m^2/s$ — $(y_c)_1 = \sqrt[3]{\frac{16^2}{9.81}} = 2.97m$

$Q = q_1\, b_1 = 16 \cdot 6 = 96\ m^3/s$ $(E_c)_1 = 1.5(y_c)_1 = 4.46m$

$q_2 = \frac{96}{b_2} = \frac{96}{4} = 24\ m^2/s$ — $(y_c)_2 = \sqrt[3]{\frac{24^2}{9.81}} = 3.89m$

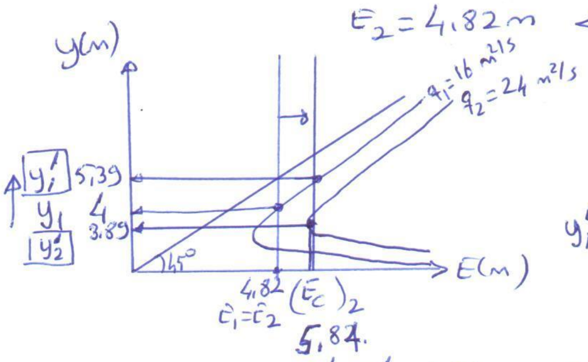
$(E_c)_2 = 1.5 \times (y_c)_2 = 5.84m$

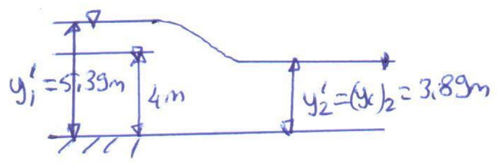
$(Fr)_1 = \frac{V_1}{\sqrt{9.81 \cdot 4}} = 0.64 < 1$

∴ subcritical flow and channel contraction there is possibility of choking. Control choking

$E_1 = E_2 \longrightarrow E_1 = y_1 + \frac{V_1^2}{2g} = 4 + \frac{16}{2 \times 9.81} = 4.82m$

$E_2 = 4.82m < (E_c)_2 = 5.84m$ ∴ choking occurs





∴ $E_1' = E_2' = 5.84m$ after choking

$E_1' = 5.84 = y_1' + \frac{q_1^2}{2g\, y_1'^2} = y_1' + \frac{16^2}{2 \times 9.81\, (y_1')^2}$

$y_1' = 5.84 - \frac{13.05}{(y_1')^2}$

y_1'(ass)	y_1'(cal)
5	5.32
5.32	5.38
5.38	5.39
5.39	$\boxed{5.39}$ ✓

∴ $y_1' = 5.39m$ after choking

$y_2' = (y_c)_2 = 3.89m$ after choking