a)Determine whether the flow is **irrotational** or not by **writing** an expression for the vorticity of the flow field described by

$V = x^2 y i - x y^2 j$

b)Consider the two dimensional flow field given by

$V = 2ti + j$

where t is time. For this flow, determine the pattern of streamlines and pathlines.

a) $\vec{V} = \underbrace{x^2 y\vec{i}}_{u} - \underbrace{x y^2}_{v}\vec{j}$

$$\vec{\nabla}\times\vec{V} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix} = i\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) - \vec{j}\left(\frac{\partial w}{\partial x} - \frac{\partial u}{\partial z}\right) + \vec{k}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

For 2D flow (only u and v)

$$\vec{\nabla}\times\vec{V} = \vec{k}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

$$\left.\begin{aligned} \frac{\partial v}{\partial x} &= -y^2 \\ \frac{\partial u}{\partial y} &= x^2 \end{aligned}\right\} \quad \vec{\nabla}\times\vec{V} = -(x^2+y^2)\vec{k} \neq 0 \quad \text{Rotational flow}$$

$$\boxed{u = \frac{dx}{dt} \qquad v = \frac{dy}{dt} \qquad w = \frac{dz}{dt}}$$

b) $V = \underbrace{2t\vec{i}}_{u} + \underbrace{\vec{j}}_{v}$

$u = 2t, \; v = 1 \Rightarrow$

Pathline

$x = \int u\,dt = \int 2t\,dt = t^2 + C$

$y = \int v\,dt = \int 1\,dt = t + C$

$$\frac{dx}{u} = \frac{dy}{v}$$

$$\frac{dx}{2t} = \frac{dy}{1} \Rightarrow \int dx = 2t\,dy$$

$x = 2ty + C$

at $t = t_0 \Rightarrow C = x_0 - 2t_0 y_0$

$x = 2ty + x_0 - 2t_0 y_0$

$$\boxed{y = (x - x_0 + 2t_0 y_0)\cdot\frac{1}{2t}}$$ Eq. of streamline

30) The velocity components of an incompressible, two-dimensional velocity field are given by the equations

$u = y^2 - x(1+x)$

$v = y(2x + 1)$

Show that the flow is irrotational and satisfies conservation of mass.

Solution:

$$\frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\vec{k} = \vec{w}_z$$

$u = y^2 - x(1+x)$

$v = y(2x + 1)$

$$\frac{\partial v}{\partial x} = 2y$$

$$\frac{\partial u}{\partial y} = 2y$$

$$\frac{1}{2}(2y - 2y) = 0 \text{ Flow is irrotational}$$

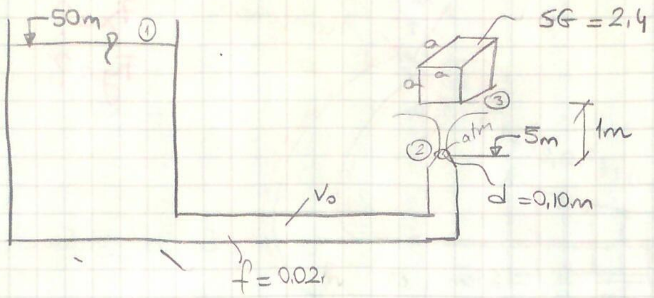
$$\vec{\nabla} \cdot \vec{V} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$$

$$\frac{\partial u}{\partial x} = -1 - 2x$$

$$\frac{\partial v}{\partial y} = 2x + 1$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = -1 - 2x + 1 + 2x = 0 \text{ Flow satisfies the conservation of mass.}$$

Exp 1



a = ?

Sln

Energy equation between ① and ②

$$\frac{\cancel{v_1^2}}{2g} + \frac{\cancel{P_1}}{\gamma} + z_1 = \frac{v_2^2}{2g} + \frac{\cancel{P_2}}{\gamma} + z_2 + f\frac{L}{D}\frac{V_0^2}{2g}$$

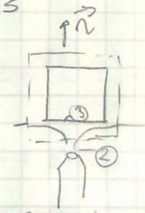
$$50 = 5 + \frac{v_2^2}{2g} + \frac{0.02 \times 250}{0,2}\frac{V_0^2}{2g} = 5 + 1,27 V_0^2 + 0,051 V_2^2$$

Continuity : $V_0 A_0 = V_2 A_2$

$$V_0 \frac{\pi 0,2^2}{4} = V_2 \cdot \frac{\pi \cdot 0,1^2}{4} \Rightarrow V_0 = 0,25 V_2$$

$$50 = 5 + 0,051 V_2^2 + 0,0794 V_2^2 \Rightarrow V_2 \cong 18,6 \, m/s$$

Energy equation between ② & ③



$$\frac{V_2^2}{2g} + \frac{P_2}{\gamma} + z_2 = \frac{V_3^2}{2g} + \frac{P_3}{\gamma} + z_3$$

$$\frac{V_2^2}{2g} = \frac{V_3^2}{2g} + 1 \Rightarrow v_3 = \sqrt{v_2^2 - 2g} = 18.06 \, m/s$$

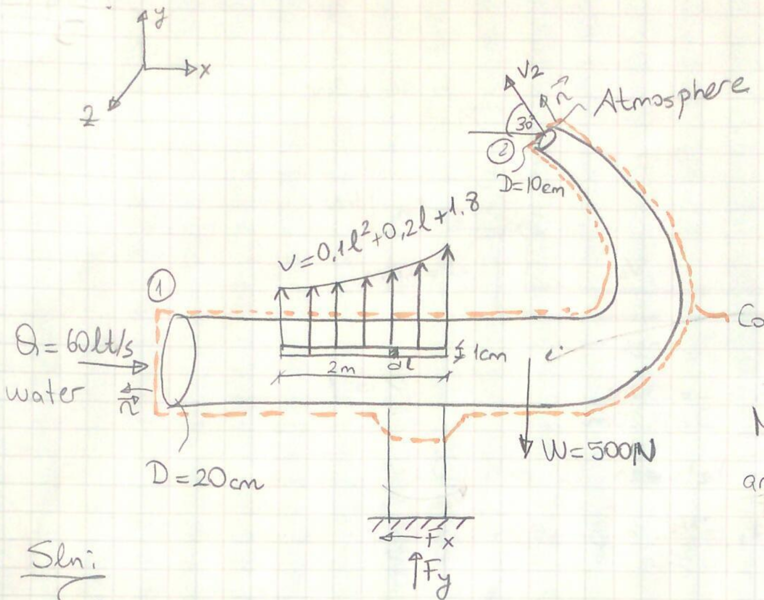
Momentum equation :

$$G = 2.4\gamma_w \cdot a^3 = 2,4 \times 9810 \, a^3 = 23544 a^3$$

$$\vec{F_y} = \oint \rho \vec{v_y} \, v_n \, dA \qquad F_y - G = 0$$

$$G = + \rho \; \overset{\oplus}{18.06} * \overset{\oplus}{18.06} \; A_3 = 1000 \times 18.06^2 \, \frac{\pi \times 0,1^2}{4} = 2561 N = 23544 a^3$$

$$\Rightarrow a \cong 0,48 m$$



$F_x = ?$

$F_y = ?$

Neglect friction losses and ~~elevation~~ differences

Sln:

Continuity :

$$\oint_{CS} \rho v_n dA = 0$$

$$-Q + \int v dA + v_2 \pi \frac{0,1^2}{4} = 0$$

$$-0,06 + \int_0^2 (0,1\ell^2 + 0,2\ell + 1,8)\underbrace{(0,01 d\ell)}_{dA} + v_2 \pi \frac{0,1^2}{4} = 0$$

$$-0,06 + \left(\frac{0,1\ell^3}{3} + \frac{0,2}{2}\ell^2 + 1,8\ell\right)(0,01)\Big|_0^2 + v_2 \pi \frac{0,1^2}{4} = 0$$

$$\Rightarrow V_2 = 2,21 m/s$$

Energy equation between 1 and 2

$$V_1 = Q/A_1 = 0,06/(\pi \times 0,2^2/4) = 1,91 m/s$$

$$\frac{v_1^2}{2g} + \frac{P_1}{\gamma} + z_1 = \frac{v_2^2}{2g} + \frac{P_2}{\gamma} + z_2$$

$$\frac{P_1}{\gamma} = \frac{v_2^2 - v_1^2}{2g} + \underbrace{(z_2 - z_1)}$$

→ Neglect this term because it is not given!

$$\frac{P_1}{\gamma} = \frac{2,21^2 - 1,91^2}{2g} \Rightarrow P_1 = 618 N/m^2$$

Momentum in X-direction

$$P_1 \pi \frac{(0,2)^2}{4} - F_x = \oint_{CS} \rho v_x v_n \, dA = -\rho \underset{v_{1x}}{\overset{(+)}{V_1}} \underset{v_{1n}}{\overset{(-)}{V_1}} \pi \frac{0.2^2}{4} - \rho \underbrace{\overset{\ominus}{V_2} \cos 30}_{V_{2x}} \underset{V_{2n}}{\overset{\oplus}{V_2}} \pi \frac{0,1^2}{4}$$

1. $$618 \, \pi \frac{0,2^2}{4} - F_x = -1000 (1,91)^2 \pi \frac{0,2^2}{4} - 1000 (2,21)^2 \cos 30 \frac{\pi \times 0,1^2}{4}$$

$\Rightarrow F_x = 167,24N$

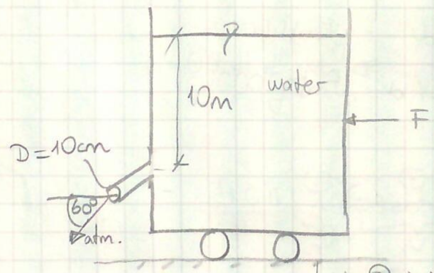
Momentum in Y-direction

$$-W + F_y = \oint_{CS} \rho v_y v_n \, dA = + \int \rho \underset{v_y}{\overset{\oplus}{v}} \underset{\hookrightarrow v_n}{\overset{\oplus}{v}} \, dA + \rho \underbrace{V_2 \sin 30}_{V_{2y}} \underset{V_{2n}}{V_2} \pi \frac{0,1^2}{4}$$

$$F_y = 500 + \rho \int_0^2 (0,1l^2 + 0,2l + 1,8)^2 \, 0,01 \, dl + \rho V_2^2 \sin 30 \, \pi \frac{0,1^2}{4}$$

$F_y = 611,29N$

Exp 3:



$\gamma_w = 9810 \, N/m^3$

$\rho_w = 1000 kg/m^3$

The weight of the tank & the water in the tank is 5000N.

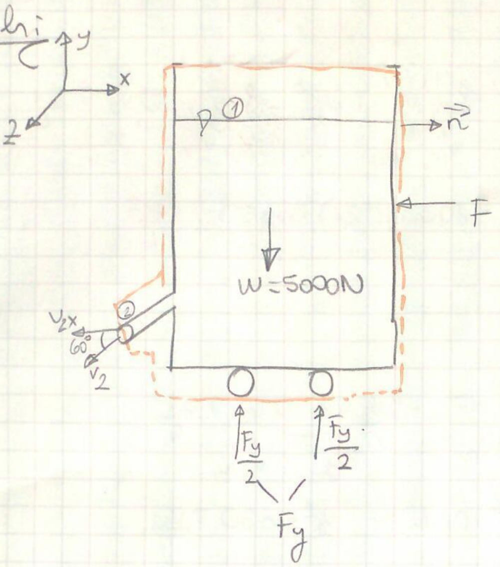
↳ Frictionless rail

a) Determine „F" horizontal force that the tank doesn't move.

b) Determine the vertical force acting on the wheel.

(Flow is steady-state)

Skh



Energy equation between ① and ②:

$$\frac{V_1^2}{2g} + \frac{P_1}{\gamma} + z_1 = \frac{V_2^2}{2g} + \frac{P_2}{\gamma} + z_2$$

$$z_1 - z_2 = \frac{V_2^2}{2g} = 10 \Rightarrow V_2 = 14.00\,m/s$$

Momentum in x-direction

$$-F = \oiint_{CS} \rho V_x V_n dA = -F = -\rho \underbrace{(V_2 \cos 60)}_{V_x}^{\ominus} \underbrace{V_2}_{V_n}^{\oplus} \pi \frac{0.1^2}{4}$$

$$F = \rho \cdot 14 \cos 60 \cdot 14\, \pi \frac{0.1^2}{4} \Rightarrow F = 769.69\,N$$

Momentum in y-direction

$$-W + F_y = \oiint_{CS} \rho V_y V_n dA = -\rho \underbrace{V_2 \sin 60}_{\ominus}^{(V_y)} \underbrace{V_2}_{\oplus}^{(V_n)} \pi \frac{0.1^2}{4}$$

$$F_y = 5000 - 1000\, 14^2 \sin 60\, \pi \frac{0.1^2}{4}$$

$$F_y = 3666.86\,N$$

1- Which one of the following sets of conditions clearly apply to an ideal fluid?

(a) Viscous and compressible

(b) Non-viscous and incompressible

(c) Non-viscous and compressible

(d) Viscous and incompressible

2- Newton's law of viscosity depends upon the

(a) Stress and strain in a fluid

(b) Shear stress, pressure and velocity

(c) Shear stress and rate of strain

(d) Viscosity and shear stress

3- What are the dimensions of kinematic viscosity of a fluid?

(a) LT^{-2}

(b) L^2T^{-1}

(c) $ML^{-1} T^{-1}$

(d)$ML^{-2} T^{-2}$

4- Decrease in temperature, in general, results in

(a) An increase in viscosities of both gases and liquids

(b) A decrease in the viscosities of both liquids and gases

(c) An increase in the viscosity of liquids and a decrease in that of gases

(d) A decrease in the viscosity of liquids and an increase in that of gases

5- Which one of the following statements is correct?

The pressure centre is:

(a) The cycloid of the pressure prism

(b) A point on the line of action of the resultant force

(c) At the centroid of the submerged area

(d) Always above the centroid of the area

6- Assertion (A): The buoyant force for a floating body passes through the centroid of the displaced volume.

Reason (R): The force of buoyancy is a vertical force & equal to the weight of fluid displaced.

(a) Both A and R are individually true and R is the correct explanation of A

b) Both A and R are individually true but R is not the correct explanation of A

(c) A is true but R is false

(d) A is false but R is true

7- The convective acceleration of fluid in the x-direction is given by:

(a) $u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}+\omega\frac{\partial \omega}{\partial z}$

(b) $\frac{\partial u}{\partial t}+\frac{\partial v}{\partial t}+\frac{\partial \omega}{\partial t}$

(c) $u\frac{\partial u}{\partial x}+u\frac{\partial v}{\partial y}+u\frac{\partial \omega}{\partial z}$

(d $u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+\omega\frac{\partial u}{\partial z}$)

8- Irrotational flow occurs when

(a) Flow takes place in a duct of uniform cross-section at constant mass flow rate.

(b) Streamlines are curved.

(c) There is no net rotation of the fluid element about its mass center.

(d) Fluid element does not undergo any change in size or shape.

9- Which one of the following is correct? The capillary rise on depression in a small diameter tube is

(a) Directly proportional to the specific weight of the fluid

(b) Inversely proportional to the surface tension

(c) Inversely proportional to the diameter

(d) Directly proportional to the surface area

10- The vertical component of force on a curved surface submerged in a static liquid is equal to the

(a) Mass of the liquid above the curved surface

(b)Weight of the liquid above the curved surface

(c) Product of pressure at C.G. multiplied by the area of the curved surface

(d) Product of pressure at C.G. multiplied by the projected area of the curved surface