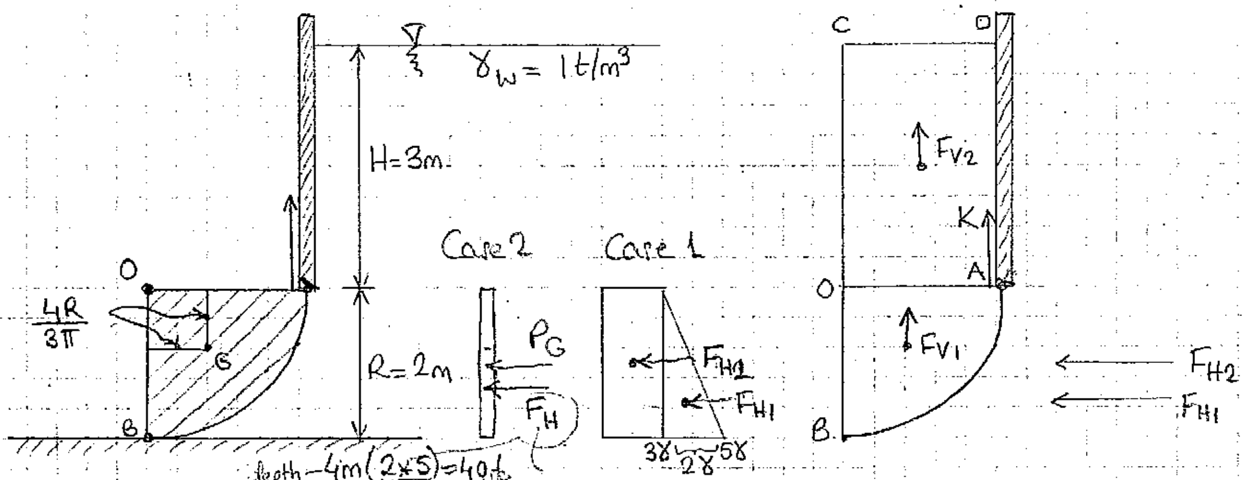
Example The gate OAB is a quarter of a cylinder with $R=2m$. and $L=5m$.

a) Find the hydrostatic force components acting on the gate.

b) What should be the force K required to open the gate if you neglect the weight of the gate.



* Note: If you find the total horizontal force acc. to Case 2 the line of action of the resultant should be calculated seperately (by using the moment inertia of rectangle)

a) $F_H = F_{H1} + F_{H2} = \frac{1}{2} 2\gamma(R)L + 3\gamma RL = \frac{1}{2} 2(1)2(5) + 3(1)2(5)$

$= 10 + 30 = 40t$

$F_V = F_{V_1} + F_{V_2} = \gamma \forall_{OAB} + \gamma \forall_{OADC} = \left(\frac{\pi R^2}{4} L + RHL\right)\gamma$

$= \left(\frac{\pi 2^2}{4} 5 + 2(3)5\right) 1 = 15.71 + 30 = 45.71\ t.$

b) Moment around point O

$K.R + F_{V_1} \frac{4R}{3\pi} + F_{V_2} \frac{R}{2} - F_{H_1} \frac{2}{3} R - F_{H2} \frac{1}{2} R = 0$

$K = 10\frac{2}{3} + 30\frac{1}{2} - \underbrace{15.71}_{\frac{\pi R^2 L}{4}} \frac{4}{3\pi} - 30\frac{1}{2} = \frac{20}{3} + \frac{30}{2} - \left(\frac{\pi 4 5}{4} \frac{4}{3\pi} + 15\right)$

$= \frac{130}{6} - \left(\frac{20}{3} + 15\right) = 0$

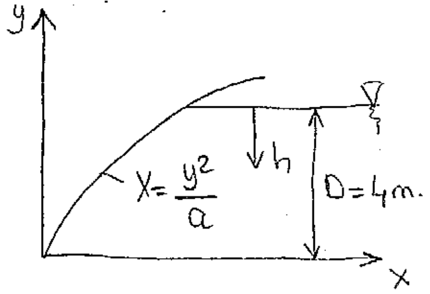
Line of action of
Pressure forces acting on a radial gate should pass through the center O. Therefore, moment around O is zero.

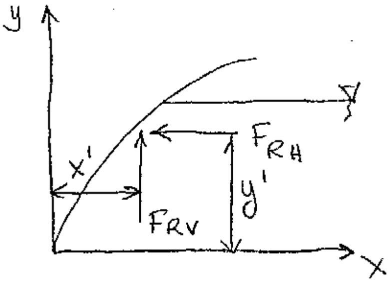
Example 1: (Intr. to Fluid Mechanics; Fox) page 66:

Given: Gate of constant width, $w = 5m$.

Equation of surface in xy plane is $x = y^2/a$, where $a = 4m$.

Water stands at depth $D = 4m$ to the right of the gate.

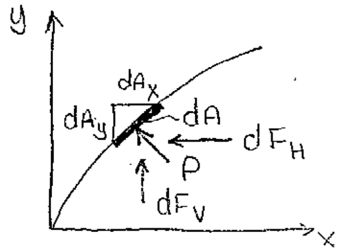




$\rho_w = 999\ kg/m^3$

$g = 9.81\ m/sec^2$

Find: F_{RH}, F_{RV} and line of action of each.



$$\left.\begin{array}{l} dF_H = P\,dA_y = \gamma h w\,dy \\ dF_V = P\,dA_x = \gamma h w\,dx \end{array}\right\} h(y) = (D-y)$$

$$\cancel{d}F_H = \int_0^D \gamma h w\,dy = \gamma w \int_0^D (D-y)\,dy \quad \leftarrow 392\text{ kN.}$$

$$\cancel{d}F_V = \int_0^{y^2/a \to D^2/a} \gamma h w\,dx = \gamma w \int_0^{D^2/a} (D-y)\,dx = \gamma w \int_0^{D^2/a} D - \sqrt{a}\,x^{1/2}\,dx \quad \leftarrow 261\text{ kN}$$

$$y' F_{RH} = \int y\,dF_H = \int y \gamma h w\,dy = \gamma w \int_0^D (D-y)\,y\,dy$$

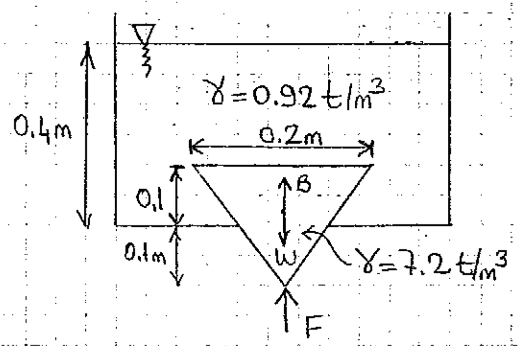
moment around z-axis

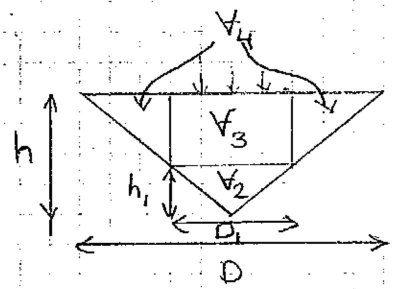
$$y' = \frac{\gamma w}{F_{RH}} \int (D-y)\,y\,dy \quad \leftarrow 1.33\text{ m.}$$

$$x' F_{RV} = \int_0^{y^2/a} dF_V\,x = \int_0^{D^2/a} \gamma h w\,dx\,x = \gamma w \int_0^{D^2/a} (D - \sqrt{a}\,x^{1/2})\,x\,dx \quad \leftarrow 1.2\text{ m.}$$

moment around z-axis.

Example Calculate the force required to lift the conical body inside the container.





$$\forall_T=\forall_1=\frac{1}{3}\frac{\pi}{4}D^2h=\frac{1}{3}\frac{\pi}{4}(0.2)^2(0.2)=2.094\times10^{-3}\,m^3$$

Dry volume of the cone;

$$\forall_2=\frac{1}{3}\frac{\pi}{4}(D_1)^2h_1=\frac{1}{3}\frac{\pi}{4}(0.1)^2(0.1)=0.262\times10^{-3}\,m^3$$

$$\forall_3=(0.1)\frac{\pi}{4}(0.1)^2=0.786\times10^{-3}\,m^3$$

$$\forall_4=\forall_1-\forall_2-\forall_3=1.046\times10^{-3}\,m^3$$

Weight of the cone $W=\gamma\forall_1=(7.2)(2.094\times10^{-3})=0.01508\,t$

Bouyoncy $B=\gamma\forall_4=(0.92)(1.046\times10^{-3})=0.00096\,t$

Pressure over volume $\forall_3\Rightarrow F_p=\gamma(0.4-0.1)\frac{\pi}{4}(D_1)^2$

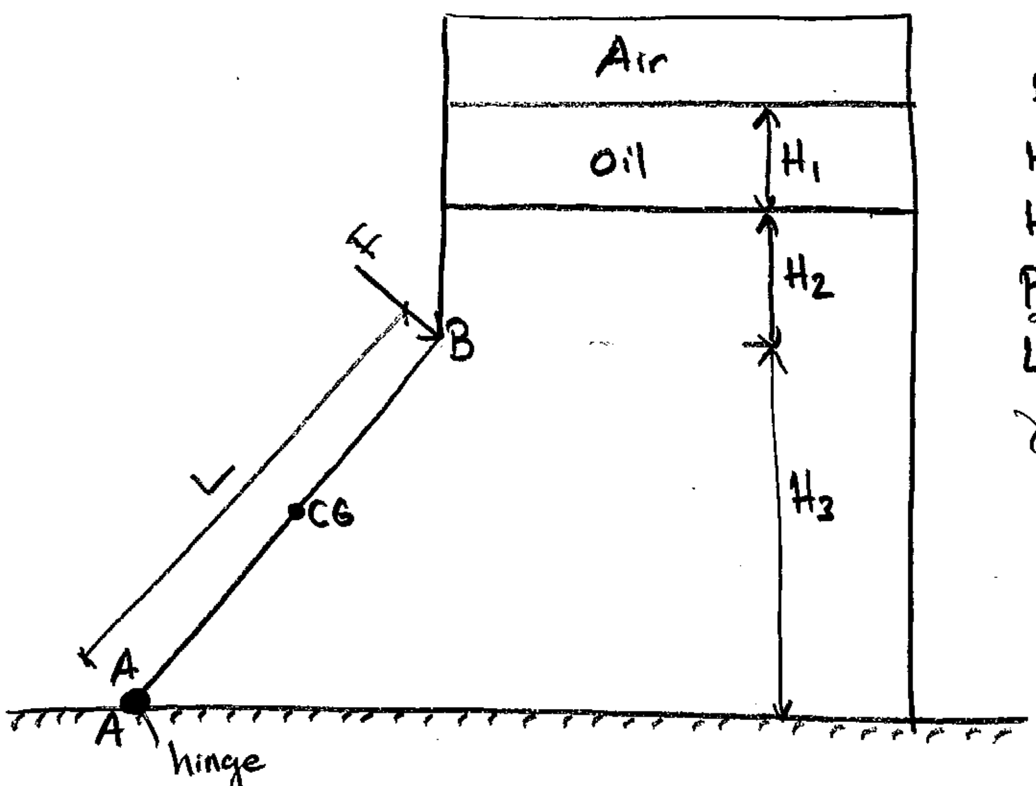
$$F_p=0.92(0.4-0.1)\frac{\pi}{4}(0.1)^2=0.00217\text{ ton}$$

Force required to lift the conical body.

$$F=W+F_p-B=0.01508+0.00217-\cancel{0.001046}\,0.00096=0.01629\,t$$

$$=16{,}29\text{ kgf.}$$

Ex:



$S.G_{oil} = 0.8$

$H_1 = 3m$

$H_2 = 2.6m$

$H_3 = 4m$

$P_{air\,gage} = 49.05\ kPa$

$L = 5m$

$\gamma_w = 9810\ N/m^3$

Calculate the force F necessary to hold the 4m-wide gate.

$F_1 = 49050 \times 5 \times 4 = 981\ kN$

$F_2 = (3 \times 0.8 \times 9810 + 2.6 \times 9810) \times 5 \times 4 = 981\ kN$

$F_3 = \frac{1}{2}\, 4 \times 9810 \times 5 \times 4 = 392.4\ kN$

$F = F_1 + F_2 + F_3 = 981 + 981 + 392.4 = 2354.4\ kN$

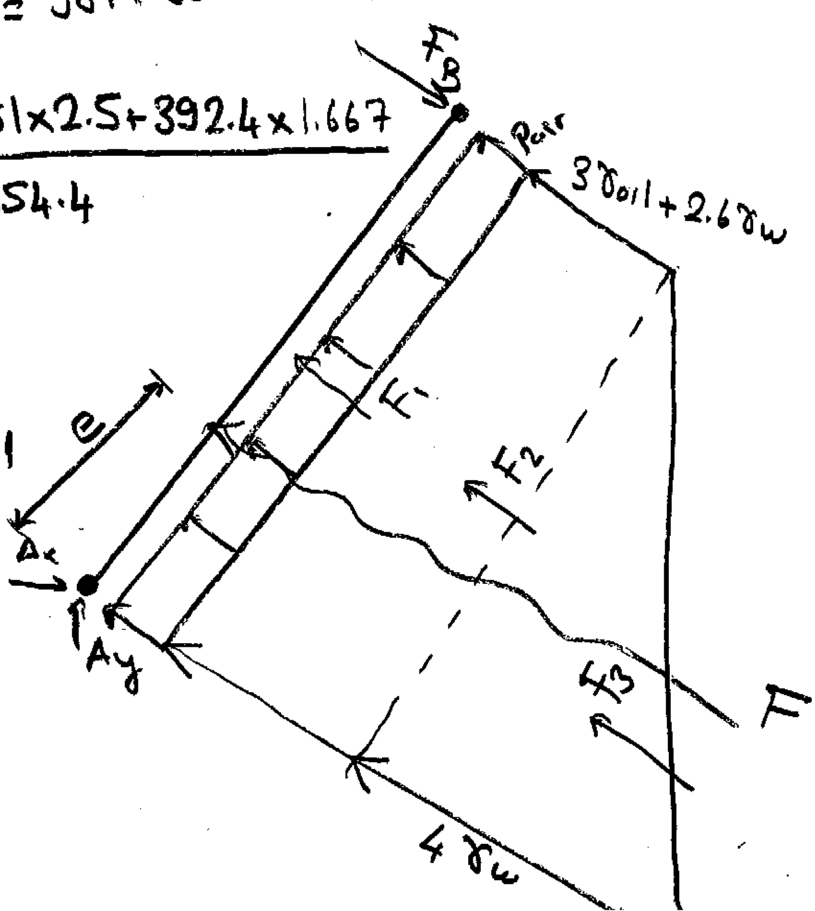
$$e = \frac{981 \times 2.5 + 981 \times 2.5 + 392.4 \times 1.667}{2354.4}$$

$e = 2.361m$

$\Sigma M_A = 0$

$F_B \times 5 = 2354.4 \times 2.361$

$F_B = 1111.75\ kN$

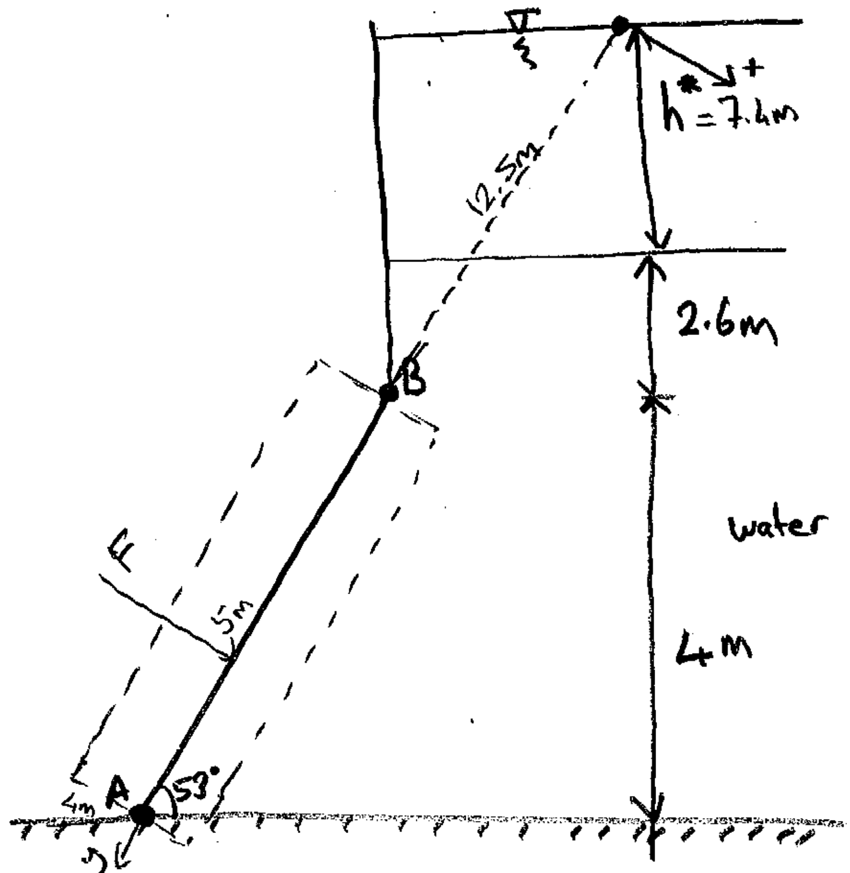


II. way of solution

$$F = P_{CG} \cdot A$$

$$P_{CG} = P_{air} + 3\gamma_{oil} + 2.6\gamma_w + 2\gamma_w$$
$$= 49.05 + 3\times0.8\times9.81 + 2.6\times9.81 + 2\times9.81 = 117.72 \text{ kN/m}^2$$

$$F = 117.72\times5\times4 = 2354.4 \text{ kN}$$



$$h^* = \frac{P_{air} + 3\gamma_{oil}}{\gamma_w} = \frac{49.05 + 3\times0.8\times9.81}{9.81} = 7.4m$$

$$y_{cp} = y_{cG} + \frac{I_{xc}}{y_{cG} \cdot A}$$; $y_{cG} = 12.5 + 2.5 = 15m$

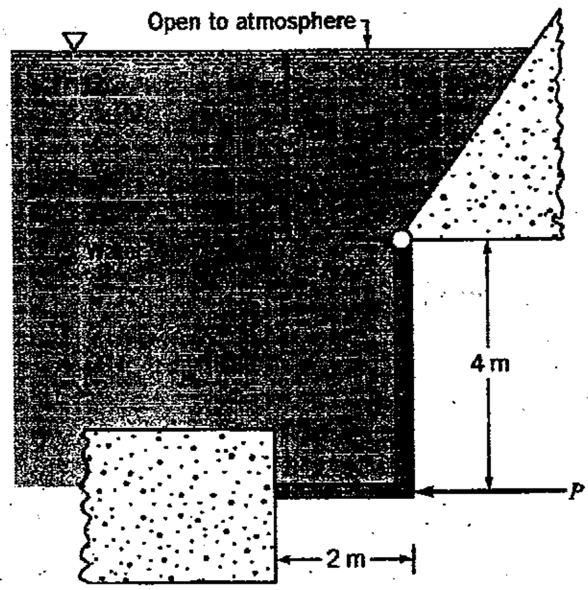
$$I_{xc} = \frac{1}{12}\times4\times5^3 = 41.667 m^4$$

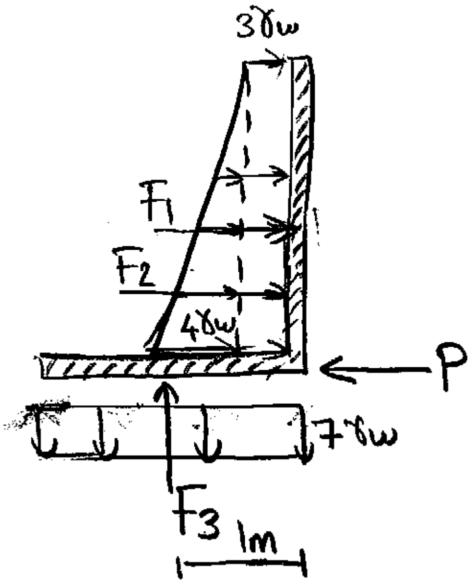
$$y_{cp} = 15 + \frac{41.667}{15\times5\times4} = 15.139m$$

$$\Sigma M_A = 0 \;:\; F_B\times5 = F\times(17.5 - 15.139)$$

$$F_B = 1111.75 \text{ kN}$$

Ex: The rigid gate OAB is hinged at O and rests against a rigid support at B. What minimum horizontal force P, is required to hold the gate closed if its width is 3 m? Neglect the weight of the gate and friction in the hinge. The back of the gate is exposed to the atmosphere.





$F_1 = 3\gamma_w \times 4 \times 3 = 36\gamma_w$

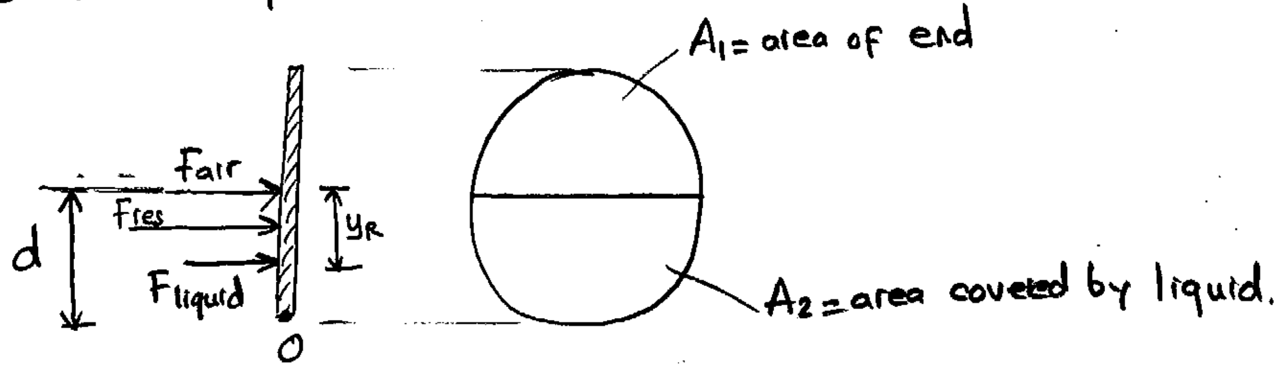
$F_2 = \frac{1}{2} \times 4\gamma_w \times 4 \times 3 = 24\gamma_w$

$F_3 = 7\gamma_w \times 2 \times 3 = 42\gamma_w$

$$\Sigma M_o = 0 \Rightarrow 36\gamma_w \times 2 + 24\gamma_w \times \frac{2}{3} \times 4 + 42\gamma_w \times 1 = 4P$$

$$\Rightarrow P = 436.5 \text{ kN}$$

Ex: A horizontal 2-m-diameter conduit is half filled with a liquid (S.G=1.6) and is capped at both ends with plane vertical surfaces. The air pressure in the conduit above the liquid surface is 150 kPa. Determine the resultant force of the fluid acting on one of the end caps, and locate this force relative to the bottom of the conduit.



$F_{air} = pA_1$, where p is air pressure

Thus, $F_{air} = 150 \times 1000 \times \frac{\pi}{4} \times 2^2 = 150\pi \times 10^3$ N

$F_{liquid} = \gamma h_c A_2$ where $h_c = \frac{4R}{3\pi}$

Thus,

$F_{liquid} = 1.6 \times 9810 \times \frac{4}{3} \times \frac{1}{2} \times \frac{\pi}{4} \times 2^2 = 10.5 \times 10^3$ N

For liquid,

$y_R = \frac{I_{xc}}{y_c A_2} + y_c$ where $I_{xc} = 0.1098\, R^4$

and $y_c = h_c = \frac{4R}{3\pi}$

Thus, $y_R = \frac{0.1098}{\frac{4}{3\pi} \times \frac{1}{2} \times \frac{\pi}{4} \times 2^2} + \frac{4}{3} = 0.5891$ m

Since $F_{resultant} = F_{air} + F_{liquid} = (150\pi + 10.5) \times 10^3 = 482$ kN

We can sum moments about "O" to locate resultant to obtain

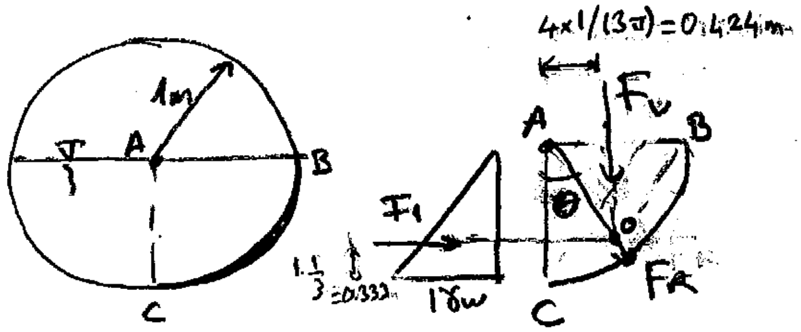
$F_{resultant} \times d = F_{air} \times 1 + F_{liquid} \times (1 - 0.5891)$

$d = \frac{(150\pi \times 10^3) \times 1 + 10.5 \times 10^3 \times 0.4109}{482 \times 10^3}$

d = 0.987 m above the bottom of conduit.

Example

The 2 m diameter drainage conduit is half full of water at rest. Determine the magnitude and line of action of the resultant force that the water exerts on a 1 m length of the curved section



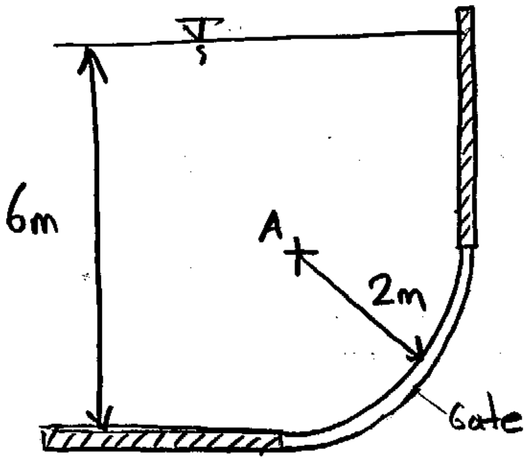
$$F_1 = F_H = \frac{1}{2}\gamma_w \times 1 \times 1 \times 1 = 4905 \text{ N}$$

$$F_V = \frac{\pi . 1^2}{4} . 1 \times \gamma_w = 7704.7 \text{ N}$$

$$F_R = \sqrt{F_V^2 + F_H^2} = 9133.5 \text{ N}$$

$$\tan\theta = \frac{F_H}{F_V} = \frac{4905}{7704.7} = 0.636 \Rightarrow \theta = 32.5^\circ$$

Ex: A 4m-long-curved gate is located in the side of a reservoir containing water as shown in the figure. Determine the magnitude of the horizontal and vertical components of the force of the water on the gate. Will this force pass through point A? Explain.



For equilibrium,

$$\Sigma F_x = 0$$

or

$$F_H = F_2 = \gamma h_{c2} A_2 = \gamma (4+1)(2\times4) = 9.80\times5\times8 = 392\ kN$$

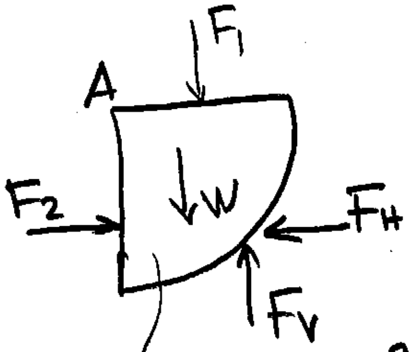
Similarly

$$\Sigma F_y = 0$$

$$F_V = F_1 + N$$

$$F_1 = (\gamma 4)\times 2\times 4 = 9.8\times4\times4$$

$$W = \gamma \forall = 9.80\times 4\pi$$



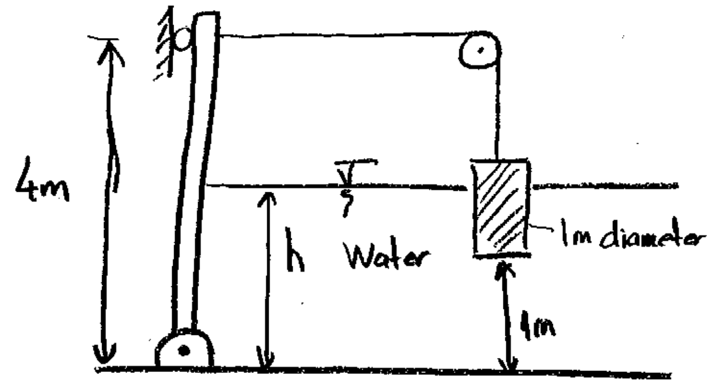
Volume $= \forall = \frac{\pi\times2^2}{4}\times4 = 4\pi$

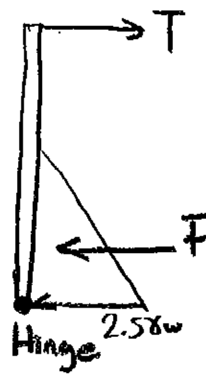
Thus,

$$F_V = 9.80\,(32+4\pi) = 437\ kN$$

Note: The direction of all differential forces acting on the curved surface is perpendicular to surface and therefore the resultant must pass through the intersection of all these forces which is at point A.

1 m diameter cylindrical mass, M is connected to a 3m wide rectangular gate. The gate is to open when the water level, h, drops below 2.5m. Determine the required value for M. Neglect friction at the gate hinge and the pulley.

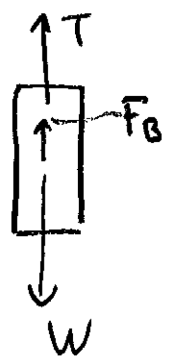




$$F = \frac{1}{2} \cdot 2.5\gamma_w \cdot 2.5 \cdot 3 = 9.375\gamma_w$$

$$\sum M_{hinge} = 0 : \quad T \cdot 4 = F \cdot \frac{2.5}{3}$$

$$\Rightarrow T = 1.953\gamma_w$$



$$T + F_B = W$$

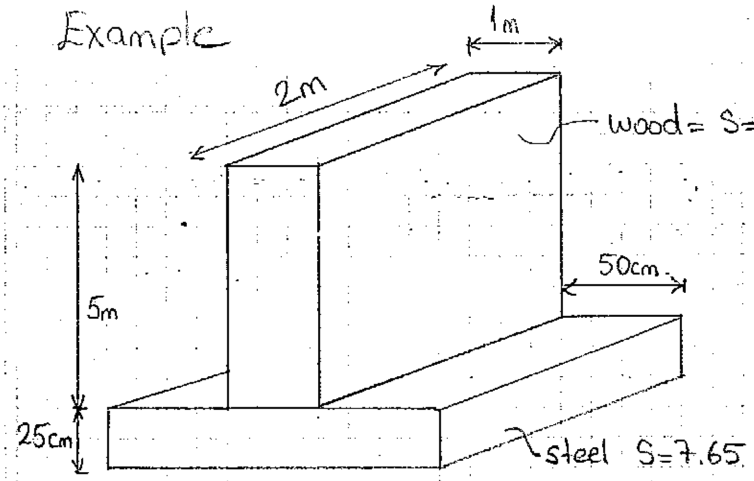
$$F_B = \frac{\pi \cdot 1^2}{4} \cdot (2.5-1)\gamma_w = 1.178\gamma_w$$

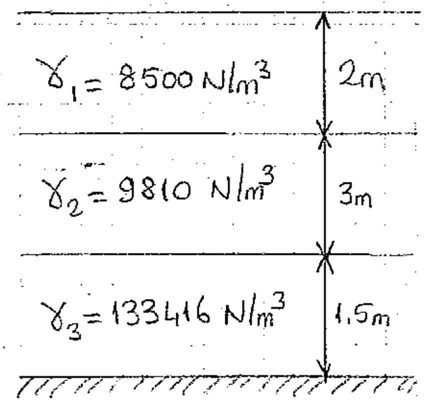
$$1.953\gamma_w + 1.178\gamma_w = W$$

$$W = 3.131\gamma_w = 30716\,N$$

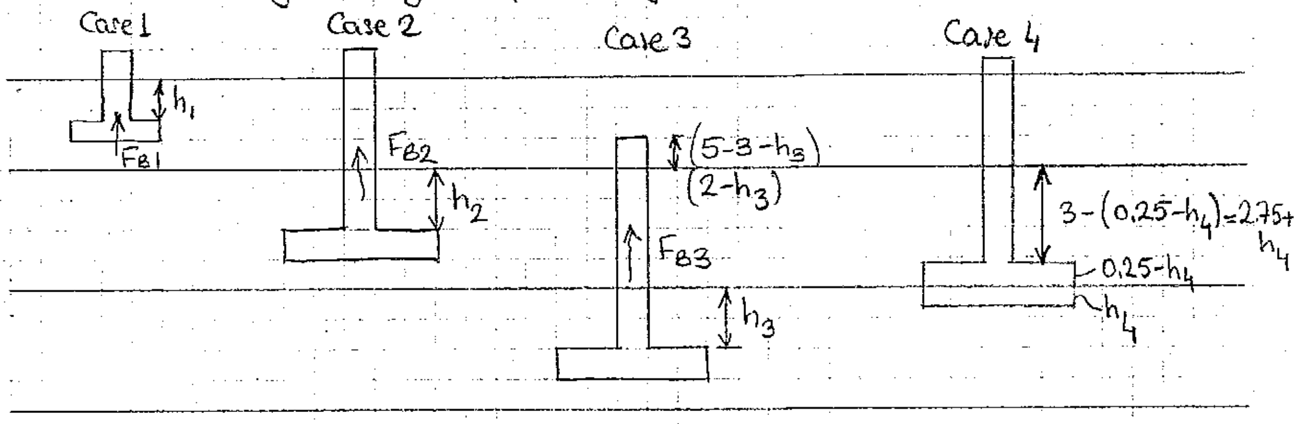
$$W = M \cdot g \Rightarrow M = \frac{30716}{9.81} = 3131\,kg$$

Example





Find the submerged height of the object.



$$W_{object} = 2(0.25)(0.5+0.5+1)*7.65(9810)+(5\times2\times1)*0.65\times9810 = 138811\ N$$

Case I : $F_{B1} = 0.25*(0.5+0.5+1)*2*8500 + h_1*(1*2)*8500 = 138811\ N$

$h_1 = 7.66\ m.$ (Physically impossible → object will sink more)

Case II : $F_{B2} = 0.25*(0.5+0.5+1)*2*9810 + h_2*(1*2)*9810 + 1\times2*2*8500$

$= 138811$

$h_2 = 4.84\ m.$

$h_2 + 0.25 = 4.84+0.25 = 5.09m > 3m.$ (Greater than height of the 2nd layer)

The object will continue to sink

Case III $F_{B3} = 0.25*(0.5+0.5+1)*2*133416 + h_3*(1*2)*133416$

$+ 3*1*2*9810 + (1*2)(2-h_3)*8500 = 138811\ N$

$h_3 = -0.35\ m.$

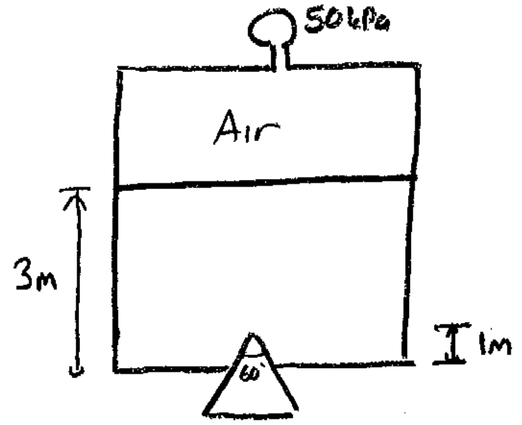
∴ Therefore, the object will float according to a case in between Case II and Case III

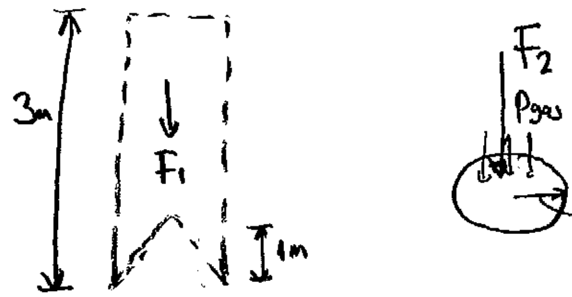
Case IV $F_{B4} = (2)(2)h_4\,133416 + (2.75+h_4)(2\times1)\,9810 + (0.25-h_4)*2*2*9810 + (2\times1)\times2*8500 = 138811$

$h_4(533664) + 87955 + 19620h_4 + 9810 - 39240h_4 = 138811$

$h_4 = 0.0798 \cong 7.98\ cm$

A plug in the bottom of a pressurized tank is conical in shape. The air pressure is 50 kPa and $\gamma_{fluid} = 27\ kN/m^3$. Determine the magnitude, direction and line of action of the force exerted on the curved surface of the cone.





$$F_1 = \left(\pi . 0.577^2 . 3 - \frac{1}{3}\pi . 0.577^2 . 1\right) . 27 = 75.306\ kN$$

$$F_2 = 50 . \pi \times 0.577^2 = 52.3\ kN$$

$$F_v = F_1 + F_2 = 75.306 + 52.3 = 127.6\ kN$$

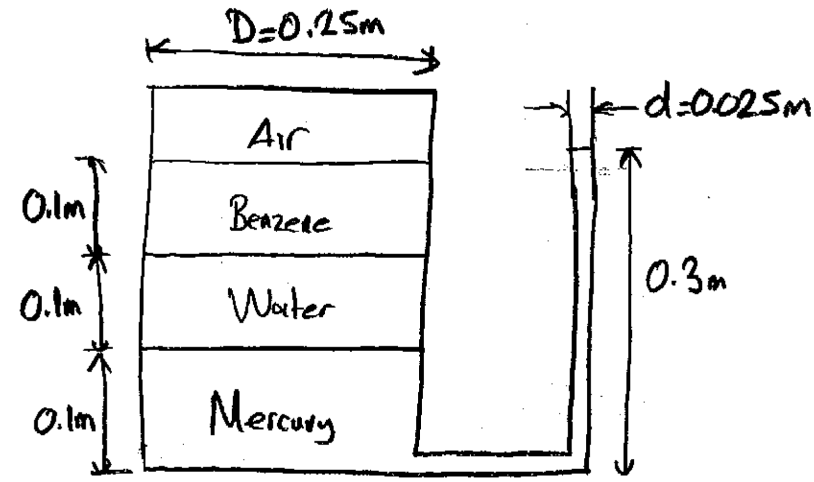
This force is directed vertically downward along the cone axis.

Consider a tank containing mercury, water, benzene and air. Find the relative air pressure. If an openning is made in the top of the tank, find the equilibrium level of the mercury in the manometer.

$SG_{mercury} = 13.55$

$SG_{benzene} = 0.879$

$\gamma_w = 9.80\ kN/m^3$



a) $P_{air_{rel}} + 0.1\gamma_b + 0.1\gamma_w + 0.1\gamma_m - 0.3\gamma_m = 0$

$P_{air_{rel}} = 0.2 \times 13.55 \times 9.80 - 0.1 \times 9.80 - 0.1 \times 0.879 \times 9.80$

$P_{air_{rel}} = 24.7\ kPa$

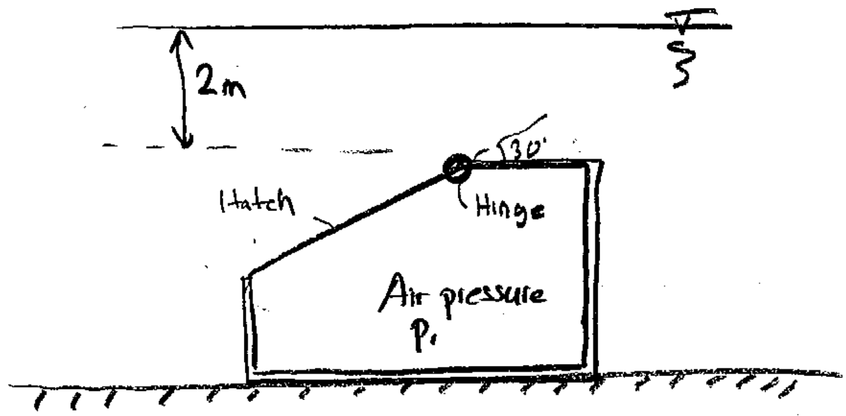
b) $0.1\gamma_b + 0.1\gamma_w + \left[0.1 + \frac{(0.3-h)\cdot\pi\cdot 0.025^2}{\pi \times 0.25^2}\right]\gamma_m = h\gamma_m$

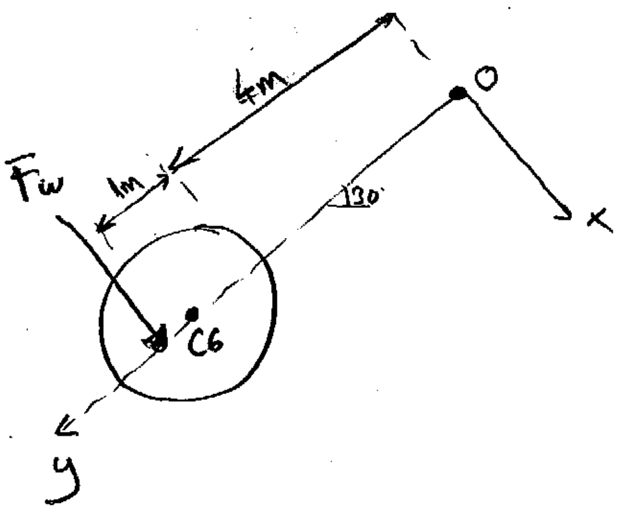
$0.1 \times 0.879\gamma_w + 0.1\gamma_w + 0.1 \times 13.55\gamma_w + 0.003 \times 13.55\gamma_w = (h + 0.01h) \times 13.55\gamma_w$

$\Rightarrow h = 0.116\ m$

A structure is attached to the ocean floor. 2m diameter hatch is located and inclined and hinged at one edge. Determine the minimum air pressure, P_1, within the container to open the hatch. Neglect the weight of the hatch and friction in the hinge.

$\gamma_{water} = 10.1\ kN/m^3$





$$F_{water} = \frac{\pi . 2^2}{4} . (5 . \sin 30) . \gamma_w = 79.325\ kN$$

$$y_{cp} = y_{CG} + \frac{I_{xxc}}{y_{CG} . A} = 5 + \frac{\pi . 1^4/4}{5 . \pi . 1^2} = 5.05m$$

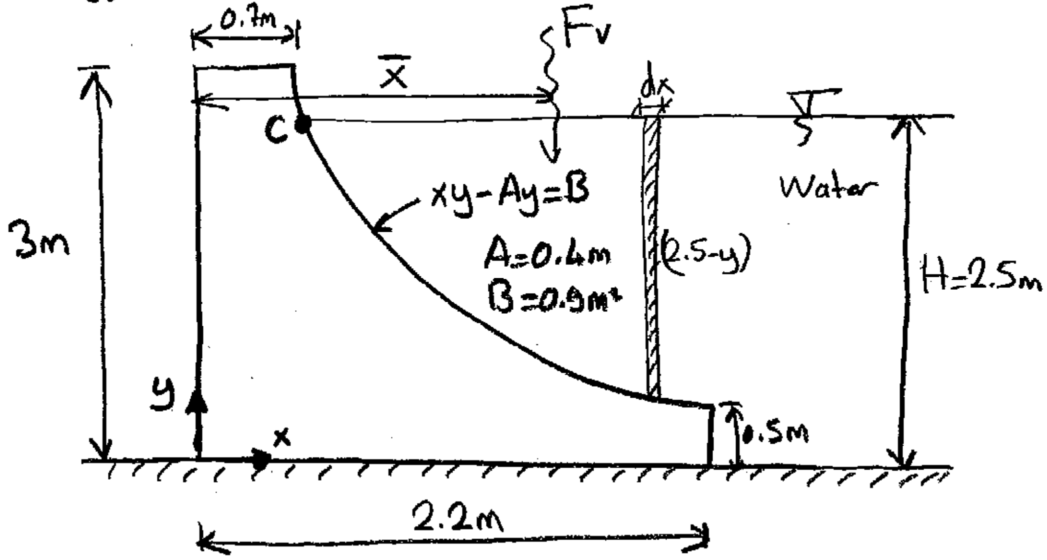
For equilibrium

$$F_w . (5.05 - 4) = P_1 . \pi . 1^2$$

$$P_1 = \frac{79.325 \times 10^3 \times 1.05}{\pi \times 1^2} = 26.5\ kPa$$

A dam is to be constructed across a river using the cross section below, Assume the dam width $w = 50$m. For water height $H = 2.5$m, calculate the magnitude and line of action vertical force of water on the dam face

$\gamma_{water} = 9810 \ N/m^3$



$$xy - 0.4y = 0.9 \Rightarrow y = \frac{0.9}{x-0.4}$$

$$y = 2.5\text{m} \Rightarrow 2.5 = \frac{0.9}{x_c - 0.4} \Rightarrow x_c = 0.76\text{m}$$

$$F_v = 50\gamma_w \int_{0.76}^{2.2} (2.5-y)\,dx = 50 \times 9810 \int_{0.76}^{2.2} \left(2.5 - \frac{0.9}{x-0.4}\right) dx$$

$$F_v = 50 \times 9810 \left[2.5x - 0.9 \ln|x-0.4|\right]_{0.76}^{2.2}$$

$$F_v = 50 \times 9810 \,(4.971 - 2.819) = 1.05 \times 10^6 \text{ N}$$

$$\bar{x} \cdot F_v = 50 \times 9810 \int_{0.76}^{2.2} x \cdot (2.5-y)\,dx$$

$$\bar{x} = \frac{50 \times 9810}{1055556} \int_{0.76}^{2.2} \left(2.5x - \frac{0.9x}{x-0.4}\right) dx$$

$$\bar{x} = 0.46468 \left[2.5\frac{x^2}{2} - 0.9\left((x-0.4) + 0.4 \ln|x-0.4|\right)\right]_{0.76}^{2.2} = 1.605\text{m}$$