CE 371 HOMEWORK 5.1

1) The velocity field $\vec{V} = ax\vec{i} - by\vec{j}$, where a = b = 1 s^{-1}, can be interpreted to represent flow in a corner.
a) Show that particle motion is described by the parametric equations $x_p = c_1 e^{at}$, $y_p = c_2 e^{-bt}$
b) Obtain equation of pathline for located at (1,2) at t = 0
c) Compare pathline with streamline through the same point.

2) A three dimensional velocity field is given by u = 2x, v = -y and w = z. Determine the acceleration vector.

3) A velocity field is given by $u = cx^2$ and $v = cy^2$, where c is a constant.**a)** Determine the x and y components of the acceleration.**b)** At what point (points) in the flow field is acceleration is zero?

4) The x and y components of a velocity field is given by $u = x^2y$ and $v = -xy^2$. Determine the equation of streamlines and pathlines of this flow.

5) The velocity field of a flow is given by $u = -V_0 y/\left(x^2 + y^2\right)^{1/2}$ and $v = V_0 x/\left(x^2 + y^2\right)^{1/2}$ where V_0 is constant. **a)** Where in the flow field is the speed equal to V_0 ? **b)** Determine the equation of the streamlines.

6) a) Show that the streamlines for a flow whose velocity components are $u = c\left(x^2 - y^2\right)$ and $v = -2xcy$, where c is a constant, are given by the equation $x^2y - y^3/3 = \text{constant}$. **b)** At which point (points) is the flow parallel to the y axis ? At which point (points) is the fluid stationary.

7) The x and y components of a velocity field are given by u = x-y and $v = x^2y-8$. Determine the location of any stagnation points in the flow field. That is, what point(s) is the velocity zero?

8) The velocity field of a flow is given by $\vec{V} = 20y/\left(x^2 + y^2\right)^{1/2}\vec{i} - 20x/\left(x^2 + y^2\right)^{1/2}\vec{j}$ m/s, where x and y in meters. **a)** Determine the fluid speed at points along the x axis; along the y axis. **b)** What is the angle between the velocity vector and the y axis at points (x,y)= (5,0), (5,5) and (0,5) ?

ANSWERS

1) b) xy = 2 (pathline); **c)** xy = 2 (streamline)
2) $\vec{a} = 4x\vec{i} + y\vec{j} + z\vec{k}$
3) a) $a_x = 2c^2x^3$, $a_y = 2c^2y^3$; **b)** (x,y) = (0,0)
4) xy = constant
5) a) $V = V_0$ for the entire flow field; **b)** $x^2 + y^2$ = constant
6) b) This occurs when $x = \pm y$; **c)** (x,y) = (0,0)
7) (x,y) = (2,2)
8) a) V = 20 m/s, **b)** θ = -90° for (5,0); θ = -45° for (5,5); θ = 0° for (0,5)

CE 371 HOMEWORK 5.2

Q1) If the velocity field is $\tilde{V} = (-2x^2 + 3y)\vec{i} + (5yz^2)\vec{j} + (5z^2t)\vec{k}$, find the acceleration components of the particle for t = 2 seconds, x = 1m, y=1m and z=1m. Also find the magnitude of acceleration and velocity.

$a_x = 8x^3 - 12xy + 15yz^2$ $|a_x| = 11\,m/s^2$ $\vec{V}_A = 1\vec{i} + 5\vec{j} + 10\vec{k}$

$a_y = 25yz^4 + 50z^3yt$ $|a_y| = 125\,m/s^2$ $|\vec{V}_A| = 11.225\,m/s$

$a_z = 50z^3t^2 + 5z^2$ $|a_z| = 205\,m/s^2$

Q2) The path of a fluid particle is given by xy=36, while at any time, the position is given by $x=6t^2$. What are the velocity and acceleration components of x and y directions ?

$\vec{a} = 12\vec{i} + \left(-\frac{432}{x^2} + \frac{10368}{x^3}t^2\right)\vec{j}$; $\vec{V} = 12t\vec{i} - \frac{432}{x^2}t\vec{j}$

Q3) The velocity field of a two dimensional flow is given by $\tilde{V} = \frac{x}{t}\vec{i} + 2y\vec{j}$ and particles pass through $A_0(x_0, y_0, z_0)$ at time t_0.

a) Find the equation of the pathline $y = y_0 e^{2\left(\frac{x}{x_0} - 1\right)t_0}$

b) Compute the equation of the streamlines.

$y = \frac{x^{2t}}{x_0^{2t}} y_0$

Q4) The velocity field of a flow is given by $\tilde{V} = 3x\vec{i} - 2y\vec{j}$

a) Find the equation of pathlines on x-y plane. $x^2y^3 = C$

b) Determine the equation of streamlines on x-y plane for the point B(3,1) $x^2y^3 = 9$

Q5) Velocity components of the flow is given by $u=a(2bt^2+y)$, v=2bt and w=0 where a and b are the constants in Eulerian description. Find the position vector of the flow in Lagrangian description for C(1,1,1), t=1 seconds.

$\vec{r} = \left[\frac{5}{3}ab t^3 + a(1-b)t + \left(1 - \frac{2}{3}ab - a\right)\right]\vec{i} + \left[bt^2 + (1-b)\right]\vec{j} + C_3\vec{k}$

CE 371 HOMEWORK 5.3

Q1) (a) If the velocity field is $\vec{V} = (3x+2y)\vec{i} + (2z+3x^2)\vec{j} + (5t-2z^2)\vec{k}$, then determine the velocity and its magnitude at point (1,3,1) for t=3s and find the acceleration vector for t= 5s at A(1,1,1). $\vec{V} = 9\vec{i} + 5\vec{j} + 13\vec{k}$; $\vec{a} = 25\vec{i} + 76\vec{j} - 87\vec{k}$

(b) If the velocity vector is $\vec{V} = 3x\vec{i} + 3y\vec{j} - 2z\vec{k}$, find the velocity and acceleration vectors and their magnitudes at point P(1,1,2). Determine the equation of streamline and pathline passing through point P. $\vec{V} = 3\vec{i} + 3\vec{j} - 4\vec{k}$; $\vec{a} = 9\vec{i} + 9\vec{j} + 9\vec{k}$

$y = x$ (streamline and pathline)

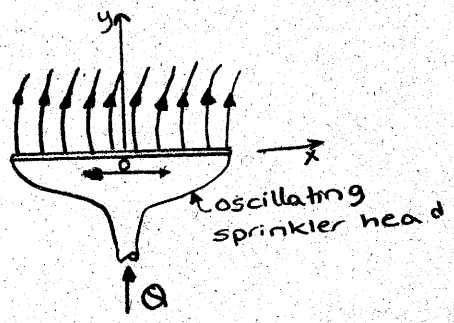
Q2) The velocity field of water flow is given by $\vec{V} = (3u_0 v_0 t^2 - v_0 y)\vec{i} - 2v_0 y\vec{j}$ where u_0 and v_0 are the constants in Eulerian description. Find the position vector of the flow in Lagrangian description for P(1,2,1), t=2 seconds.

$\vec{r} = [u_0 v_0 t^3 - 2v_0 e^{v_0(4-2t)} t + 1 + 4v_0 - 8u_0 v_0]\vec{i} + 2e^{v_0(4-2t)}\vec{j} + 1\vec{k}$

Q3) The path of a fluid particle is given by xy=16, while at any time, the position is given by $x=4t^2$. What are the velocity and acceleration components of x and y directions ?

$\vec{V} = 8t\vec{i} - \frac{8}{t^2}\vec{j}$; $\vec{a} = 8\vec{i} + \frac{24}{t^4}\vec{j}$

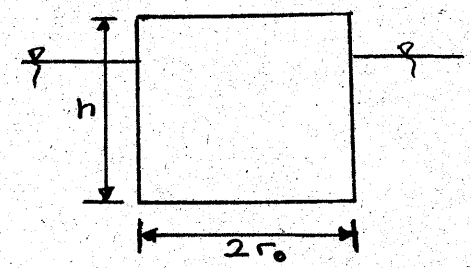
Q4) Water flowing from the oscillating slit shown in the figure produces a velocity field given by $\vec{V} = u_0 \sin[\omega(t - y/v_0)]\vec{i} + v_0\vec{j}$ where u_0, v_0 and ω are constants. (a)Determine the streamline that passes through the origin at t=0 and at $t=\pi/2\omega$ (b)Determine the pathline of the particle that was at the origin at t=0 and at $t=\pi/2\omega$



a) $x = \frac{u_0}{\omega}\sin\left(\frac{\omega y}{v_0}\right)$

b) $y = \frac{v_0}{u_0}x$

(Canceled) **Q5)** What are the proportions of radius to height (r_0/h) of a right-circular cylinder of specific gravity SG so that it will float in water with end faces horizontal in stable equilibrium?



NOTE: For every question, major steps of calculation haveto be shown. Otherwise no mark will be given even if the answer is correct.

CE 371 HOMEWORK 5.4

1) The three components of velocity in a flow field are given by,

$u = x^2 + y^2 + z^2$

$v = xy + yz + z^2$

$w = -3xz - z^2/2 + 4$

a) Is this flow incompressible? **b)** Determine an expression for the rotation vector. Is this an irrotational flow field?

2) Determine an expression for the rotation of the flow described by,

$\mathbf{V} = -4xy^3\mathbf{i} + y^4\mathbf{j}$

Is the flow irrotational?

3) For what combination of constants a, b, c and e can the velocity components,

$u = ax + by$

$v = cx + ey$

$w = 0$

be used to describe an incompressible flow field ?

4) The velocity components of an incompressible, two dimensional velocity field are given by the equations,

$u = 2xy$

$v = x^2 - y^2$

Show that the flow is irrotational and satisfies the conservation of mass.

5) The x component of velocity in steady, incompressible flow field in xy plane is u = A/x, where A = 2 m^2/s, and x is measured in meters. Find the simplest y component of velocity for this flow field.

ANSWERS

1) a) Yes; **b)** $w = -(y/2 + z)i + (5z/2)j - (y/2)k$ **;** Rotational flow field

2) $\mathbf{w} = 6xy^2\mathbf{k}$; Rotational flow field

3) $a + e = 0$

5) $v = Ay/x^2$