A comparative performance analysis of endoreversible dual cycle under maximum ecological function and maximum power conditions

Bahri Şahin a, Osman Azmi Özsoysal b, Oğuz Salim Söğüt b, a

Department of Naval Architecture, Yıldız Technical University, Beşiktaş, 80750, Istanbul, Turkey
b Faculty of Naval Architecture and Ocean Engineering, Istanbul Technical University, Maslak, 80626, Istanbul, Turkey

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Abstract

In this paper, a performance analysis and optimization based on the ecological criterion has been performed for an air-standard endoreversible internal combustion engine dual cycle coupled to constant temperature heat reservoirs. The ecological objective function, defined as the power output minus the loss rate of availability is taken as the optimization criterion. The optimal performances and design parameters, such as compression ratio, pressure ratio, cut-off ratio and NTU allocation ratio, which maximize the ecological objective function are investigated. The obtained results are compared with those of the maximum power performance criterion. Since the ecological optimization technique for a dual cycle consists of both power and entropy generation rate, the obtained results lead more realistic design from the point of view of preservation of natural resources.

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1. Introduction

In the last two decades, by using finite time thermodynamics theory, many optimization studies based on various performance criteria have been carried out for endoreversible and irreversible heat engine models. The reader interested in these optimization works could refer to literature surveys written by Bejan [1] and Chen et al. [2]. In these studies, the performance of heat engines has been analyzed for the power, thermal efficiency, specific power, power density, entropy generation, thermo-economics and ecological objectives by taking into account finite rate external heat transfer irreversibility and/or internal irreversibilities. During the last decade, some authors have studied the ecological performance of endoreversible and irreversible heat engines by considering finite-time and finite-size constraints [3–8]. The ecological objective function introduced by Angulo-Brown [3] and improved by Yan [4], is defined by

\[ E = W - T_0 \dot{S}_g, \]

where \( W \) is the power output, \( \dot{S}_g \) is the entropy generation rate and \( T_0 \) is the environment temperature. Using the ecological optimization criterion, Angulo-Brown [3] investigated optimal performance conditions for the endoreversible Carnot heat engine. He found that the thermal efficiency at maximum ecological objective function condition is in good agreement with the arithmetic mean of the Carnot efficiency and the maximum power efficiency. Furthermore, he showed that the use of the maximum ecological objective function gives about 80% of the maximum power output but with an entropy production of only 30% of the entropy that would be produced by the maximum power output condition. It should be noted that maximization of the ecological objective function represents the best compromise between the power output, \( W \) and the loss rate of availability \( T_0 \dot{S}_g \). The ecological performance analysis, first introduced by Angulo-Brown [3] has been extended to an irreversible Carnot heat engine model coupled to variable temperature heat reservoirs by Cheng and Chen [8]. They investigated the optimal design parameters that maximize the ecological objective function. In this perspective, they showed that the heat conductance of the hot-end heat exchanger should be smaller than that of the cold-end heat exchanger from the viewpoint of ecological objective. The ecological optimization technique has been applied to the endoreversible and irreversible Brayton cycle coupled to constant-temperature heat reservoirs by Cheng and Chen [6, 7]. They investigated the optimal performances and design parameters under the maximum ecological objective function conditions, and then they examined the effects of the number of transfer units of heat exchangers, reservoir tem-
temperature ratio, turbine and compressor isentropic efficiencies and heat leak on the global and optimal performances.

Much interest has been recently paid to the optimization of air-standard reciprocating heat engine cycles, i.e., Otto, Diesel and Dual cycles, based on maximum power objective [9–16]. A performance analysis on Otto, Diesel and Dual cycles using finite-time ecological optimization technique does not appear to have been published. Application of the maximum ecological objective function to performance optimization of the endoreversible dual cycle seemed to be interesting since the preservation of natural resources may be considered in the analysis as well as power output.

2. Theoretical analysis

A schematic illustration of an endoreversible dual cycle (1-2-3-4-5-1) coupled to constant temperature heat reservoirs is shown in Fig. 1. Process 1-2 is an isentropic compression. The heat addition occurs in two steps: processes 2-3 and 3-4 are constant volume and constant pressure heat addition processes respectively. Process 3-4 also makes up the first part of the power stroke. The isentropic expansion process, 4-5 is the remainder of the power stroke. A constant volume heat rejection process, 5-1, completes the cycle. In the cycle, \( \dot{Q}_{H1} \) and \( \dot{Q}_{H2} \) are the rates of heat transfer from the hot reservoir at temperature \( T_H \) to working fluid in processes 2-3 and 3-4, \( \dot{Q}_L \) is the rate of heat transfer from the working fluid to the cold reservoir at temperature \( T_L \) in process 5-1. Since processes 2-3, 3-4 and 5-1 occur across finite temperature differences, the dual cycle is externally irreversible, although it is internally reversible. Assuming the working fluid is an ideal gas with constant specific heat, the rates at which heat is supplied, \( \dot{Q}_{H1} \) and \( \dot{Q}_{H2} \), and rejected \( \dot{Q}_L \), are given by:

![Fig. 1. P–V and T–S diagrams for the Dual cycle.](image-url)
\[ \dot{Q}_{H1} = U_{H1} A_{H1} \frac{(T_H - T_2) - (T_H - T_3)}{\ln[(T_H - T_2)/(T_H - T_3)]} \]
\[ = \frac{1}{k} C_W(T_3 - T_2) \quad (1) \]

\[ \dot{Q}_{H2} = U_{H2} A_{H2} \frac{(T_H - T_3) - (T_H - T_4)}{\ln[(T_H - T_3)/(T_H - T_4)]} \]
\[ = C_W(T_4 - T_3) \quad (2) \]

\[ \dot{Q}_L = U_L A_L \frac{(T_3 - T_L) - (T_1 - T_L)}{\ln[(T_3 - T_L)/(T_1 - T_L)]} \]
\[ = \frac{1}{k} C_W(T_5 - T_1) \quad (3) \]

where \( U_{H1} A_{H1} \) and \( U_{H2} A_{H2} \) are the hot-side heat exchanger conductance and \( U_L A_L \) is the cold-side heat exchanger conductance, \( C_W \) is the capacitance rate of the working fluid and \( k \) is the isentropic exponent. From Eqs. (1), (2) and (3),

\[ \dot{Q}_{H1} = \frac{1}{k} C_W \varepsilon_{H1}(T_H - T_2) = \frac{1}{k} C_W(T_3 - T_2) \quad (4) \]

\[ \dot{Q}_{H2} = C_W \varepsilon_{H2}(1 - \varepsilon_{H1})(T_H - T_2) = C_W(T_4 - T_3) \quad (5) \]

\[ \dot{Q}_L = \frac{1}{k} C_W \varepsilon_L(T_3 - T_L) = \frac{1}{k} C_W(T_5 - T_1) \quad (6) \]

and the effectivenss of the hot-side heat exchanger \( \varepsilon_{H1} \) and \( \varepsilon_{H2} \), and cold-side heat exchanger \( \varepsilon_L \) for counterflow heat exchangers are defined as:

\[ \varepsilon_{H1} = 1 - \exp(-N_{H1}) \quad (7) \]

\[ \varepsilon_{H2} = 1 - \exp(-N_{H2}) \quad (8) \]

\[ \varepsilon_L = 1 - \exp(-N_L) \quad (9) \]

where the number of heat transfer units based on the minimum thermal capacitance rates are:

\[ N_{H1} = \frac{k U_{H1} A_{H1}}{C_W} \]
\[ N_{H2} = \frac{U_{H2} A_{H2}}{C_W} \]
\[ N_L = \frac{k U_L A_L}{C_W} \]

Eqs. (4), (5) and (6) give:

\[ T_3 = \varepsilon_{H1} T_H + (1 - \varepsilon_{H1}) T_2 \quad (10) \]
\[ T_4 = [\varepsilon_{H1} + \varepsilon_{H2}(1 - \varepsilon_{H1})]T_H + (1 - \varepsilon_{H1})(1 - \varepsilon_{H2})T_2 \]
\[ T_1 = \varepsilon_L T_L + (1 - \varepsilon_L) T_5 \quad (12) \]

From the first law of thermodynamics, the power output is given by:

\[ \dot{W} = (\dot{Q}_{H1} + \dot{Q}_{H2}) - \dot{Q}_L \]
\[ = C_W \left[ \frac{1}{k} \varepsilon_{H1} + \varepsilon_{H2}(1 - \varepsilon_{H1}) \right] (T_H - T_2) \]
\[ - C_W \frac{1}{k} \varepsilon_L (T_5 - T_L) \quad (13) \]

From Eq. (13),

\[ T_3 = T_L + \frac{\varepsilon_{H1} + k \varepsilon_{H2}(1 - \varepsilon_{H1})}{\varepsilon_L} T_H \]
\[ - \frac{\varepsilon_{H1} + k \varepsilon_{H2}(1 - \varepsilon_{H1})}{\varepsilon_L} T_2 = \frac{k \dot{W}}{C_W \varepsilon_L} \quad (14) \]

Substituting Eq. (14) into Eq. (12) yields:

\[ T_1 = T_L + \frac{(1 - \varepsilon_L)(\varepsilon_{H1} + k \varepsilon_{H2}(1 - \varepsilon_{H1}))}{\varepsilon_L} T_H \]
\[ - \frac{(1 - \varepsilon_L)(\varepsilon_{H1} + k \varepsilon_{H2}(1 - \varepsilon_{H1}))}{\varepsilon_L} T_2 = \frac{k(1 - \varepsilon_L) \dot{W}}{C_W \varepsilon_L} \quad (15) \]

From the second law of thermodynamics, the following equation is obtained:

\[ T_1 T_3^{1-k} T_4^k - T_2 T_3 = 0 \quad (16) \]

Substituting Eqs. (10), (11), (14) and (15) into Eq. (16) gives:

\[ \dot{W} = C_W \varepsilon_L T_H \left[ \frac{1}{k} \left( \frac{T_2 a_6 - a_{4a}}{T_2 - (1 - \varepsilon_L)a_4} \right) \right] \quad (17) \]

where

\[ a_1 = \varepsilon_{H1} + k \varepsilon_{H2}(1 - \varepsilon_{H1}) \]
\[ a_2 = \varepsilon_{H2}(1 - \varepsilon_{H1}) + \varepsilon_{H1} \]
\[ a_3 = (1 - \varepsilon_{H1})(1 - \varepsilon_{H2}) \]
\[ a_4 = [\varepsilon_{H1} T_H + (1 - \varepsilon_{H1})T_2]^{1-k}[a_2 T_H + a_3 T_2]^k \]
\[ a_5 = \frac{T_L}{T_H} + (1 - \varepsilon_L) a_1 \left( 1 - \frac{T_2}{T_H} \right) \]
\[ a_6 = \frac{T_L}{T_H} + a_1 \left( 1 - \frac{T_2}{T_H} \right) \]

The objective function for ecological optimization proposed by Angulo-Brown [3] and modified by Yan [4] is:

\[ E = \dot{W} - T_0 S_g \quad (18) \]

where \( T_0 \) is the environment temperature and \( S_g \) is the entropy generation rate. Substituting Eqs. (4), (5), (6) and (14), (15) into Eq. (18) yields:

\[ E = \dot{W} + T_0 \left[ \frac{\dot{Q}_L}{T_L} - \frac{\dot{Q}_{H1}}{T_H} - \frac{\dot{Q}_{H2}}{T_H} \right] \]

or

\[ E = \dot{W} \left( 1 + \frac{T_0}{T_L} \right) - T_0 \frac{1}{k} C_W \varepsilon_L a_1 \left( \frac{1}{T_L} - \frac{1}{T_H} \right) (T_H - T_2) \quad (19) \]

The thermal efficiency may, then be expressed as:

\[ \eta = \frac{T_H[(T_2 a_6 - a_{4a})/(T_2 - (1 - \varepsilon_L)a_4)]}{a_1(T_H - T_2)} \quad (20) \]

A special case may be considered, when

\[ N_{H1} \to \infty: \varepsilon_{H1} = 1 \]
\[ N_{H2} \to \infty: \varepsilon_{H2} = 1 \]
\[ N_L \to \infty: \varepsilon_L = 1 \]
which corresponds to the classical reversible Otto cycle. The heat transfer rates are,
\[ \dot{Q}_{H1} = \frac{1}{k} \dot{C}_W(T_H - T_2) \]  
(21)
\[ \dot{Q}_{H2} = 0 \]  
(22)
\[ \dot{Q}_L = \frac{1}{k} \dot{C}_W(T_L - T_1) \]  
(23)
and since, \( a_1 = 1, \ a_2 = 1, \ a_3 = 0 \)
\[ \dot{W} = \frac{1}{k} \dot{C}_W \left( T_L + T_H - T_2 - \frac{T_1 T_H}{T_2} \right) \]  
(24)

Differentiation with respect to \( T_2 \) and seeking for a maximum yields:
\[ \frac{\partial \dot{W}}{\partial T_2} = 0 \]  
(25)
\[ T_2 = \sqrt{T_H T_L} \]  
(26)
The thermal efficiency at maximum power condition can be derived as:
\[ \eta = 1 - \frac{T_1}{T_2} \]  
(27)
Substituting \( T_2 \) into Eq. (27) yields the thermal efficiency at maximum power condition,
\[ \eta_{mp} = 1 - \frac{T_1}{T_H} \]  
(28)
which is the thermal efficiency at maximum work condition of a classical reversible Otto cycle [17], and is also well known as the Curzon–Ahlborn efficiency.

The ecological function can be expressed as:
\[ E = \frac{2}{k} \dot{C}_W \left( \frac{T_L + T_H - T_2}{T_2} - \frac{T_L T_H}{T_2} \right) \]  
(29)
\[ - \frac{1}{k} \dot{C}_W \left( 1 - \frac{T_L}{T_H} \right) (T_H - T_2) \]  
(29)
Differentiation with respect to \( T_2 \) and seeking for a maximum yields:
\[ \frac{\partial E}{\partial T_2} = 0 \]  
(30)
\[ T_2 = \frac{T_H}{2} \sqrt{\frac{2 T_L}{T_H + T_L}} \]  
(31)
Substituting \( T_2 \) into Eq. (27) gives the thermal efficiency at maximum ecological function condition:
\[ \eta_{opt} = 1 - \left( \frac{T_L}{2 T_H} + \frac{T_L}{T_H} \right)^{1/2} \]  
(32)
which is the same equation reported by Angulo-Brown [18] for an endoreversible cycle.

### 3. Results and discussion

To see the advantages and disadvantages of the design at maximum ecological function conditions (MEF), detailed numerical examples are provided and compared with those of the maximum power objective (MP). In the numerical calculations, we assume that the total number of heat transfer units is constrained \((N_H + N_L = \text{constant})\), \(N_H = N_{H1} + N_{H2}\) and \(N_{H1} = N_{H2}\).

The variation of non-dimensional ecological function with non-dimensional power output for a dual cycle is illustrated in Fig. 2. The effects of \( \chi = N_H/(N_H + N_L) \), and \( \tau = (T_H/T_L) \), on the ecological performance are shown in Fig. 2(a) and (b), respectively. It is observed that a decrease in \( \chi \) and \( \tau \) yields a significant decrease on the global and optimal performance. In addition, the power output at MEF condition is less than the MP output. For the conditions considered, the ecological function and the power output reach their maximums at \( \chi = 0.5 \). This situation will be addressed again later in the discussion.

Fig. 3 shows the variation of non-dimensional ecological function with thermal efficiency. The effects of \( \chi \) and \( \tau \) on the ecological performance and the thermal efficiency are shown in Fig. 3(a) and (b), respectively. For the practical values of the ecological function, i.e., \( E > 0 \), non-dimensional ecological function increases as \( \chi \) is increased, but the thermal efficiency at MEF \((\eta_{opt})\) is not affected by \( \chi \). Fig. 3(a). However, it may be observed in Fig. 3(b) that as \( \tau \) is increased, the maximum value of the ecological function and the thermal efficiency at MEF condition increase.

The effects of \( \chi \) and \( \tau \) on the global and optimal performance with respect to power output and thermal efficiency are shown in Fig. 4(a) and (b). The power output exhibits a rising characteristic, as \( \chi \) and \( \tau \) both increase. Although, the thermal efficiency at MP condition is not affected by \( \chi \), it increases with \( \tau \).

The effect of NTU allocation ratio, \( \chi \) on the ecological function and power output is also investigated in detail, Fig. 5. It is observed that the optimum value of \( \chi \) is 0.5 and it is independent of \( \tau \).

Compression ratio, \( r = V_1/V_2 = (T_2/T_1)^{1/(k-1)} \), pressure ratio, \( \beta = T_3/T_2 \) and cut-off ratio, \( \rho = T_4/T_3 \) are the principal engine design parameters. Therefore, the determination of the optimal values of these parameters with respect to ecological function and power output is important. An analysis is carried out and the results for compression ratio, pressure ratio and finally cut-off ratio are presented in Figs. 6 and 7, Figs. 8 and 9 and Figs. 10 and 11, respectively.

It may be seen in Figs. 6 and 7 that as \( \chi \) and \( \tau \) both increase, the compression ratio at MEF condition, \( r_{opt} \) and the compression ratio at MP condition, \( r_{mp} \) both increase. In addition, \( r_{opt} \) is found greater than \( r_{mp} \).

Pressure ratio and cut-off ratio at MEF condition, \( \beta_{opt} \) and \( \rho_{opt} \), and their corresponding values at MP condition, \( \beta_{mp} \) and \( \rho_{mp} \) and their variation with \( \chi \) and \( \tau \) are illustrated in Figs. 8–11. It is observed that as \( \chi \) increases, \( \beta_{opt} = \rho_{opt} = \beta_{mp} = \rho_{mp} \), but as \( \tau \) increases, \( \beta_{opt} = \rho_{opt} \) decrease.

### 3.1. Numerical examples

The effects of non-dimensional ecological function on the optimal performance are illustrated in Figs. 9 and 10. It is observed that as \( \chi \) increases, the optimal performance increases. However, as \( \tau \) increases, the optimal performance decreases. This situation will be addressed again later in the discussion.
Fig. 2. Non-dimensional ecological function vs. non-dimensional power output with $T_0 = T_L$; (a) $\tau = 8$: $\chi = 0.05, 0.25, 0.50$ and 0.60; (b) $\chi = 0.50$: $\tau = 4, 6, 8$ and 10.

Fig. 3. Non-dimensional ecological function vs. thermal efficiency with $T_0 = T_L$; (a) $\tau = 8$: $\chi = 0.05, 0.25, 0.50$ and 0.60; (b) $\chi = 0.50$: $\tau = 4, 6, 8$ and 10.
Fig. 4. Non-dimensional power output vs. thermal efficiency with $T_0 = T_L$; (a) $\tau = 8$: $\chi = 0.05, 0.25, 0.50$ and 0.60; (b) $\chi = 0.50$: $\tau = 4, 6, 8$ and 10.

Fig. 5. (a) Non-dimensional ecological function vs. $\chi$ with $T_0 = T_L$ and $\tau = 4, 6, 8$ and 10; (b) Non-dimensional power output vs. $\chi$ with $T_0 = T_L$ and $\tau = 4, 6, 8$ and 10.
Fig. 6. Non-dimensional ecological function vs. compression ratio with $T_0 = T_L$; (a) $\tau = 8$: $\chi = 0.05, 0.25, 0.50, 0.60$; (b) $\chi = 0.50$: $\tau = 4, 6, 8$ and 10.

Fig. 7. Non-dimensional power output vs. compression ratio with $T_0 = T_L$; (a) $\tau = 8$: $\chi = 0.05, 0.25, 0.50, 0.60$; (b) $\chi = 0.50$: $\tau = 4, 6, 8$ and 10.
Fig. 8. Non-dimensional ecological function vs. pressure ratio ($\beta$) with $T_0 = T_L$; (a) $\tau = 8$: $\chi = 0.05, 0.25, 0.50$ and 0.60; (b) $\chi = 0.50$: $\tau = 4, 6, 8$ and 10.

Fig. 9. Non-dimensional power output vs. pressure ratio ($\beta$) with $T_0 = T_L$; (a) $\tau = 8$: $\chi = 0.05, 0.25, 0.50$ and 0.60; (b) $\chi = 0.50$: $\tau = 4, 6, 8$ and 10.
Fig. 10. Non-dimensional ecological function vs. cut-off ratio ($\rho$) with $T_0 = T_L$; (a) $\tau = 8$: $\chi = 0.05$, 0.25, 0.50 and 0.60; (b) $\chi = 0.50$: $\tau = 4, 6, 8$ and 10.

Fig. 11. Non-dimensional power output vs. cut-off ratio ($\rho$) with $T_0 = T_L$; (a) $\tau = 8$: $\chi = 0.05$, 0.25, 0.50 and 0.60; (b) $\chi = 0.50$: $\tau = 4, 6, 8$ and 10.
$\beta_{mp}$ and $\rho_{mp}$ increase sharply up to a certain value and then start decreasing, Figs. 8(a), 9(a), 10(a) and 11(a). It can also be clearly seen in Figs. 8(b), 9(b), 10(b) and 11(b) that as $\tau$ increases, $\beta_{opt}$, $\rho_{opt}$, $\beta_{mp}$ and $\rho_{mp}$ increase monotonically.

The optimal values of $\beta$, $\rho$ and $\tau$ with respect to ecological function and power output and also the effects of $\chi$ and $\tau$ on these parameters are presented in Figs. 12–15. Although, the optimal NTU allocation ratio at MEF and MP conditions is determined as $\chi = 0.5$ in the theoretical analysis, this ratio is less than 0.5 for the reciprocating engines due to the design constraints. This situation should be taken into consideration.

The variation of non-dimensional ecological function and non-dimensional power output with respect to entropy generation rate is illustrated in Fig. 16 for selected $\chi$ and $\tau$ values. It can be concluded from Fig. 16 that the power loss...
penalty at MEF condition is rewarded by the lower entropy generation rate.

The optimal performance parameters at MEF and MP conditions are presented in Fig. 17. The ratio of power output at MEF to maximum power, $\dot{W}_{\text{opt}}/\dot{W}_{\text{max}}$ and the ratio of entropy generation rate at MEF to MP condition, $\dot{S}_{g,\text{opt}}/\dot{S}_{g,\text{max}}$ are observed to be independent of $\chi$, Fig. 17(a) but they exhibit a rising character with $\tau$ as seen in Fig. 17(b). For practical $\tau$ values at MEF conditions, in spite of about 10% decrease in power output, a decrease in entropy generation rate of about 30% is observed. The optimal thermal efficiency, $\eta_{\text{opt}}$ at MEF and $\eta_{\text{mp}}$ at MP conditions with respect to $\tau$ are shown in Fig. 17(c). It may be seen that $\eta_{\text{opt}}$ is greater than $\eta_{\text{mp}}$ about 13% for all of the $\tau$ values considered.
Fig. 16. Non-dimensional power output and non-dimensional ecological function vs. entropy generation with $T_0 = T_L$: (a) $\tau = 8$, $\chi = 0.05$ and $0.25$; (b) $\chi = 0.50$, $\tau = 6$ and $10$.

Fig. 17. (a) $W_{opt}/W_{max}$ and $S_{g,opt}/S_{g,mp}$ vs. $\chi$ with $T_0 = T_L$ and $\tau = 4$, $6$, $8$ and $10$; (b) $W_{opt}/W_{max}$ and $S_{g,opt}/S_{g,mp}$ vs. $\tau$ with $\chi = 0.50$; (c) $\eta_{opt}$ and $\eta_{mp}$ vs. $\tau$ with $\chi = 0.50$. 
4. Conclusions

We have presented a thermo-ecological optimization to determine the optimal operation and design parameters for the endoreversible dual cycle. In this perspective, the optimum pressure ratio ($\beta$), cut-off ratio ($\rho$), compression ratio ($r$) and NTU allocation ratio ($\chi$) that maximize the ecological objective function have been investigated. The performance of the dual cycle under maximum ecological objective function is discussed and compared with respect to maximum power output conditions. It is shown that for a dual cycle, although a design based on maximum ecological objective conditions has the advantage of lower entropy generation rate and higher thermal efficiency, it has the disadvantage of lower power output than maximum power conditions. The analysis and optimization study carried out in this work are hoped to provide guidelines for optimal design in terms of entropy generation rate, power and thermal efficiency for internal combustion engines.

References